

**MQM Comprehensive Examination  
Measurement Minor Examination**

**Day 3**

Answer both of the following two questions. Begin each response on a new page, and clearly number the item to which you are responding. **You may not use books or notes when answering these questions.**

1. The item response functions for three latent trait models are shown below. First, define the term *item response function* in language suitable for a lay audience. Second, identify the name of the model that each equation represents and differentiate between these three models by explaining how they differ in terms of the assumptions or requirements they imply and the types of data for which they are appropriate.

a. 
$$\pi_{ni} = \frac{\exp(\theta_n - \delta_i)}{1 + \exp(\theta_n - \delta_i)}$$

b. 
$$\pi_{ni} = \frac{\exp[\alpha_i(\theta_n - \delta_i)]}{1 + \exp[\alpha_i(\theta_n - \delta_i)]}$$

c. 
$$\pi_{ni} = \frac{\exp \sum_{j=0}^x (\theta_n - \delta_i - \tau_{ik})}{\sum_{k=0}^{m_i} \exp \sum_{j=0}^k (\theta_n - \delta_i - \tau_{ij})}$$

2. The concept of information in item response theory is closely related to the standard error of measurement in classical test theory.
- Define information from an IRT perspective. Give a formula for information and define all of the terms used in the formula.
  - Define standard error of measurement from a classical test theory perspective. Give a formula for standard error of measurement and define all of the terms used in the formula.
  - Explain the connection between standard error of measurement and information. Provide at least one mathematical expression that shows the connection.

Answer two of the following four questions. Begin each response on a new page, and clearly number the item to which you are responding. **You may not use books or notes when answering these questions.**

3. The items and output on the two pages that follow are from an exploratory factor analysis of 19 items that appear on an instrument designed to measure an examinee's attribution of mathematics test performance, where *attribution* is defined as one's beliefs about variables that influence one's test performance. The designers of the instrument wrote items for the instrument to measure various aspects of the test environment, examinee, and test items to which test performance may be attributed.

The instrument developer chose to retain three factors based on preliminary factor analyses of these data. He also chose to perform an oblique rotation.

Address the following questions based on your knowledge of exploratory factor analysis and the output provided.

- a. Upon what basis might the analyst have based his decision to retain three factors? Cite the values of specific indices in your explanation.
- b. Is the decision to utilize an oblique rotation reasonable for these data? Cite the values of specific indices in your explanation.
- c. Provide a substantive interpretation of the extracted factors by interpreting the factor pattern and factor structure matrices in light of the item text. Be sure to identify which items define each of the factors that you interpret.

Items and SAS (proc factor) output for Item 3.

| <i>Item</i> | <i>My performance on the tests was largely influenced by ...</i>      | <i>Strongly Agree</i> | <i>Agree</i> | <i>Disagree</i> | <i>Strongly Disagree</i> |
|-------------|---|-----------------------|--------------|-----------------|--------------------------|
| 1           | guessing.   | ○                     | ○            | ○               | ○                        |
| 2           | luck (good luck or bad luck).   | ○                     | ○            | ○               | ○                        |
| 3           | my ability or inability to concentrate.                               | ○                     | ○            | ○               | ○                        |
| 4           | my ability or inability to do the kind of work required by the test.  | ○                     | ○            | ○               | ○                        |
| 5           | my attitude towards mathematics (positive or negative).               | ○                     | ○            | ○               | ○                        |
| 6           | my health today (healthy or sick).                                    | ○                     | ○            | ○               | ○                        |
| 7           | my interest or lack of interest in mathematics.                       | ○                     | ○            | ○               | ○                        |
| 8           | my knowledge of the mathematics content covered by the test.          | ○                     | ○            | ○               | ○                        |
| 9           | my knowledge or lack of knowledge about mathematics.                  | ○                     | ○            | ○               | ○                        |
| 10          | my level of confidence about mathematics tests.                       | ○                     | ○            | ○               | ○                        |
| 11          | my level of motivation (high or low).                                 | ○                     | ○            | ○               | ○                        |
| 12          | other people in the testing room.                                     | ○                     | ○            | ○               | ○                        |
| 13          | the complexity or simplicity of the test content.                     | ○                     | ○            | ○               | ○                        |
| 14          | the effort or lack of effort that I put into answering the questions. | ○                     | ○            | ○               | ○                        |
| 15          | the method used to administer the tests.                              | ○                     | ○            | ○               | ○                        |
| 16          | the presence or absence of distractions in the testing room.          | ○                     | ○            | ○               | ○                        |
| 17          | the specific topics addressed by the questions.                       | ○                     | ○            | ○               | ○                        |
| 18          | the strategies I used to solve the problems.                          | ○                     | ○            | ○               | ○                        |
| 19          | the types of questions the tests contained.                           | ○                     | ○            | ○               | ○                        |

**Eigenvalues of the Correlation Matrix**

|   | Eigenvalue | Difference | Proportion | Cumulative |
|---|------------|------------|------------|------------|
| 1 | 4.48161890 | 1.72837191 | 0.2359     | 0.2359     |
| 2 | 2.75324700 | 0.92130203 | 0.1449     | 0.3808     |
| 3 | 1.83194496 | 0.42052176 | 0.0964     | 0.4772     |
| 4 | 1.41142321 | 0.32564383 | 0.0743     | 0.5515     |
| 5 | 1.08577937 | 0.07297362 | 0.0571     | 0.6086     |
| 6 | 1.01280575 | 0.03955048 | 0.0533     | 0.6619     |
| 7 | 0.97325527 | 0.23060990 | 0.0512     | 0.7132     |

**Promax Rotation Method Inter-Factor Correlations**

|         | Factor1 | Factor2 | Factor3 |
|---------|---------|---------|---------|
| Factor1 | 1.00000 | 0.16286 | 0.12827 |
| Factor2 | 0.16286 | 1.00000 | 0.17328 |
| Factor3 | 0.12827 | 0.17328 | 1.00000 |

**Promax Rotated Factor Pattern (Standardized Regression Coefficients)**

|  | Factor1  | Factor2  | Factor3  |
|--|----------|----------|----------|
| 1 guessing   | -0.62026 | 0.28440  | -0.02534 |
| 2 luck   | -0.64988 | 0.43730  | -0.05495 |
| 3 ability to concentrate                                   | -0.02058 | 0.51383  | 0.17952  |
| 4 ability to do work required by the test                  | 0.66939  | 0.03665  | -0.07678 |
| 5 attitude towards mathematics                             | -0.07142 | 0.11341  | 0.83708  |
| 6 health today   | 0.04253  | 0.56210  | 0.10728  |
| 7 interest in mathematics                                  | -0.06517 | 0.11725  | 0.81250  |
| 8 knowledge of the mathematics content covered by the test | 0.79183  | -0.03646 | 0.16719  |
| 9 knowledge about mathematics                              | 0.74743  | 0.00562  | 0.14765  |
| 10 confidence about mathematics tests                      | 0.15746  | 0.03663  | 0.67542  |
| 11 level of motivation                                     | 0.10360  | 0.07638  | 0.64337  |
| 12 other people in the testing room                        | -0.28276 | 0.67249  | -0.03870 |
| 13 complexity of the test content                          | 0.42253  | 0.49198  | -0.05838 |
| 14 effort put into answering the questions                 | 0.26167  | 0.39493  | 0.22451  |
| 15 method used to administer the tests                     | -0.04565 | 0.50227  | 0.10591  |
| 16 presence of distractions in the testing room            | 0.01120  | 0.58196  | 0.17872  |
| 17 specific topics addressed by the questions              | 0.47685  | 0.38798  | -0.23910 |
| 18 strategies used to solve the problems                   | 0.50824  | 0.06694  | 0.15374  |
| 19 types of questions the tests contained                  | 0.59812  | 0.36179  | -0.27807 |

**Promax Factor Structure (Correlations)**

|  | Factor1  | Factor2 | Factor3  |
|--|----------|---------|----------|
| 1 guessing   | -0.57719 | 0.17899 | -0.05562 |
| 2 luck   | -0.58571 | 0.32194 | -0.06254 |
| 3 ability to concentrate                                   | 0.08613  | 0.54159 | 0.26592  |
| 4 ability to do work required by the test                  | 0.66551  | 0.13236 | 0.01544  |
| 5 attitude towards mathematics                             | 0.05443  | 0.24683 | 0.84757  |
| 6 health today   | 0.14784  | 0.58762 | 0.21014  |
| 7 interest in mathematics                                  | 0.05814  | 0.24743 | 0.82446  |
| 8 knowledge of the mathematics content covered by the test | 0.80734  | 0.12147 | 0.26244  |
| 9 knowledge about mathematics                              | 0.76729  | 0.15294 | 0.24450  |
| 10 confidence about mathematics tests                      | 0.25006  | 0.17932 | 0.70197  |
| 11 level of motivation                                     | 0.19857  | 0.20474 | 0.66990  |
| 12 other people in the testing room                        | -0.17820 | 0.61973 | 0.04156  |
| 13 complexity of the test content                          | 0.49517  | 0.55068 | 0.08108  |
| 14 effort put into answering the questions                 | 0.35479  | 0.47645 | 0.32651  |
| 15 method used to administer the tests                     | 0.04974  | 0.51319 | 0.18709  |
| 16 presence of distractions in the testing room            | 0.12890  | 0.61475 | 0.28099  |
| 17 specific topics addressed by the questions              | 0.50936  | 0.42421 | -0.11070 |
| 18 strategies used to solve the problems                   | 0.53886  | 0.17635 | 0.23053  |
| 19 types of questions the tests contained                  | 0.62137  | 0.41102 | -0.13865 |

4. Describe two standard setting methods. One of the methods you describe must be the *Angoff* method, but the other method is your choice. In your response, be sure to describe a step-by-step procedure for carrying out each method and the advantages and disadvantages of each method.
5. An educational policy maker wants to set a performance standard for the state 8<sup>th</sup> grade mathematics exam. This examination consists of multiple-choice questions.
  - a. Make a recommendation for the standard setting method that you think will work well in this situation and give the reasons for your recommendation.
  - b. Describe in detail the process for using this standard setting procedure beginning with the statement of policy and ending with the approved numerical score on the test.
6. Large scale testing programs regularly run DIF analyses as part of their quality control procedures. Respond to the following questions from the context of such testing programs.
  - a. What DIF analysis procedure would you recommend for a large scale testing program? Give the reasons for your recommendation.
  - b. What does it mean if an item is determined to have significant DIF? Give an answer that would help a policy maker understand what DIF is all about.
  - c. When would you delete an item that has a large DIF statistic from a test? Give the rationale for your answer.