THE ROLE OF A LEARNING COMMUNITY IN CHANGING PRESERVICE. TEACHERS' KNOWLEDGE AND BELIEFS ABOUT MATHEMATICS EDUCATION[1]

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Commonly, preservice elementary teachers bring to their professional studies deeply rooted ideas about the teaching and learning of mathematics. These ideas are embedded in the content knowledge, the pedagogical experiences, and the epistemological orientation of prospective teachers. They view mathematics as a fixed body of knowledge that is best learned by memorizing facts, rules and formulas, and procedures for applying them to textbook exercises. They view the role of the teacher as carrying out goals determined by text material, providing demonstrations and examples of tasks to be completed, and checking assignments for completeness and accuracy. They expect their teacher preparation program to provide the techniques to make teaching efficient and effective. This conception of mathematics education contrasts with the nature and the creation of knowledge in the discipline (Davis and Hersh, 1981), and it denies children's natural capacity for and interest in understanding mathematical ideas (Carpenter, 1985; Resnick, 1983; Riley, Greeno, and Heller, 1983; Romberg and Carpenter, 1986). Further, it conceives of teaching as a matter of technical competence rather than reflection and decision making based on what children are coming to know.

The literature on the impact of professional study on teachers' beliefs points to the difficulty in overcoming ingrained notions developed during previous school experiences (Ball, 1988; Feiman-Nemser, 1983; Tabachnick, Popkewitz, and Zeichner, 1979-80; Zeichner, Tabachnick, and Densmore, 1987). If we are to cause prospective teachers to rethink these beliefs, we must create situations where these beliefs are faced and reconsidered. This demands powerful interventions that challenge and yet are safe situations in which students can take mathematical, emotional, and intellectual risks. Creating

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Perry Lanier, professor in MSU's Department of Teacher Education, directs the project and the Academic Learning Program from which the 23 preservice elementary teachers are drawn. The Academic Learning Program is an alternative teacher education program that emphasizes the development of a thorough understanding of school subject matters and a conceptual change view of learning and teaching. Each teacher candidate in the program has a unique field experience which involves working with a mentor teacher and a classroom of children each term over a two-year period.

a community of learners with shared responsibility for learning holds the promise of providing such an environment (National Council of Teachers of Mathematics, 1989a, 1989b; Schwab, 1976).

What is the building of a community of learners likely to contribute to learning to teach mathematics? We have ample evidence that learning in isolation from interaction with others is likely to result in students' constructing mathematical worlds that have little fit with the accepted "truths" of the discipline (Erlwanger, 1973). One might extrapolate that learning to teach in isolation from the experience of personal interactions with others' exploring the discipline itself—how one learns and what it means to know and do mathematics—would lead to equally impoverished views of what it means to teach mathematics. Thus, the creation of a community in which one's private world is exposed has the potential to challenge the learner's currently held views and lead to the construction of more acceptable and powerful views. It is through the give and take—the back and forth of shared questions, ideas, and feelings—that one-to-one community begins.

The opening up of oneself to community can, as Schwab (1975) puts it, happen "in one and only one way—through speech, by talk" (p. 32). He calls this "symbolic exchange" and describes the implications for classrooms:

> It is of first importance that the pattern of classroom life be rich in occasions for this symbolic exchange among children, this sharing, through language, of things seen and significances perceived. Such symbolic sharing of experiences is the uniquely effective means by which children convey and receive recognitions of their personality, of their existence as affecting personality and affected individuals. It is also a means by which children convey to one another the bread-and-butter promise of mutual support in difficulty. It is the means, too, by which children find sources of help and occasions for the giving of help. (p. 32)

We believe that these ideas are equally valid with preservice teachers. The act of receiving help, of being nurtured, is important. But of equal importance, especially to preservice teachers, is the giving of help. However, herein lies one of the traps—distinguishing between help given by telling, which results in dependent learners, and help given by questioning and collaborating, which results in empowered learners. Lampert (1985) describes the dilemma of teaching as

> an argument between opposing tendencies within oneself in which neither side can come out the winner. From this perspective my job would involve maintaining the tension between . . . pushing students to achieve and providing a comfortable learning environment, between covering the curriculum and attending to individual understanding. (p. 183)

Preservice teachers in their own learning of mathematics and learning to teach need to confront such dilemmas. Do teachers insist on telling learners how to solve the problem or do they give them ownership of ideas by contributing prompts, questions, counterexamples to wrong directions, or strategies for thinking about the problem situation? The former moves students and teacher quickly through material, but only a few truly encounter the potential of the ideas embedded in the problem and its solution. The latter approach has the potential to open ideas up to more of the community and shape the understanding of the one who gives help. Cobb (1989) argues that "in attempting to understand the child's mathematics, the researcher frequently elaborates his or her own mathematics" (p. 32). The giver of help who seeks to understand the current conceptions of his or her students/ colleagues will also have occasion to reflect on his or her own understanding.

We have a conception of mathematicians as working on the boundaries of the discipline, actively engaged in pushing those boundaries outward. This conception works for students of mathematics as well. They, just as mathematicians, are working on the edge of their knowledge. The fact that others have previously passed this way mathematically does not take away from the excitement and the struggle that can accompany personal and group sense making in mathematics. As Cobb (1989) has argued:

> Each child can be viewed as an active organizer of his or her personal mathematical experiences and as a member of a community or group who actively contributes to the group's continued regeneration of taken-for-granted ways of doing mathematics. . . . Children also learn mathematics as they attempt to fit their mathematical actions to the actions of others and thus to contribute to the construction of consensual domains. (p. 34)

By the tasks chosen and the nature of the discourse orchestrated, a teacher surrounds students with a safety net (or a straitjacket, depending on the nature of the choices) that helps guide the creation of mathematical knowledge that reasonably fits with the "truths" of the discipline. This is the power and the awesome responsibility of the teacher in the creation of a community of learners. Cobb (1989) supports this direction:

> If we are serious about encouraging students to be mathematical meaning-makers, we should view the teacher and the students as constituting an intellectual community. The classroom setting should be designed as much as possible to allow students to do their own truth-making. This approach contrasts sharply with codified, academic formalism that, to the initiated, signify communally-sanctioned truths that have been institutionalized by others. (p. 38)

The emphasis on meaning making is key to changing the current conceptions of preservice teachers about mathematics and the learning of mathematics. If students are to build mathematical schemas they can use in a flexible way to approach new problem situations, then they must develop the disposition to seek ways to make sense of new ideas rather than use short-term memory to "survive the test." Establishing a classroom where arguments are made to support conjectures, and where the criterion for what makes sense is determined by students and teacher working together, is likely to engender in students a very different view of mathematics from the typical rule-and-procedure orientation.

Alibert (1988) provides a picture of classroom discourse that is supportive of what we believe is important for preservice teachers to experience in the making of mathematics. He argues for new customs in the classroom, the first of which is uncertainty:

> A large place must be left for uncertainty in the learning process. Uncertainty in relation to mathematical knowledge is institutionalized in the notion of conjecture, the validation of which, and even the production of which, is devolved onto the community of students. (p. 32)

Arguments about proof are made to convince other students, not simply addressed to the teacher, to show that one has organized knowledge in an acceptable way. New mathematical ideas and tools are organized in such a way that they appear to be needed to solve some perplexing problem. And, finally, reflection occurs that helps students become more consciously aware of their own knowledge. We hold these to be valid and desirable ways for children to meet mathematics. And we hold these as equally valid and desirable ways for future teachers of mathematics to meet mathematics and questions of teaching and learning mathematics.

Community has another powerful aspect to contribute to empowering future teachers. Community can be used to encourage the propensity to value mathematics, to use mathematics, and to share mathematics. We get support for these ideas from Schwab (1976). Collaborative activities can be carried out in such a fashion that the members of the community develop propensities (a) toward service and sympathy—collaborative learning, (b) toward the suspension of impulse and the valuing of different methods and points of view, (c) toward listening to, understanding, and taking responsibility for one's collaborators, and (d) toward the negotiation of what is to be allowed into the domain of working knowledge of the group.

This paper examines an intervention study in an elementary teacher education program. The intervention—a sequence of mathematics courses, a methods course, and a curriculum seminar—had as a basic goal demonstrating the feasibility of creating in new teachers a more conceptual level of knowledge about mathematics and the teaching and learning of mathematics. A central feature of the intervention was establishing a community of learners. The question this paper addresses is, *What is* *the building of a community of learners likely to contribute to learning mathematics and learning to teach mathematics?*

We first discuss the ways in which creating a learning community promoted conceptual change in our preservice teachers' beliefs about themselves as learners of mathematics, what it means to know mathematics, and how mathematics is learned. We argue that this intervention made a significant contribution to empowering prospective elementary teachers as *learners of mathematics*.

We move then to a discussion of the implications of this intervention for teaching mathematics to children. We provide brief cases of two of our graduates in their first year as classroom teachers to illustrate the complexity of connecting their own experiences as learners of mathematics to new visions of the mathematics classrooms they might construct for young learners. We conclude with a set of questions that emerges from our research findings, a set of questions of interest and urgency for all mathematics educators attempting to transform elementary mathematics education by reforming preservice teacher education.

The Intervention

The 23 students studied by the Elementary Mathematics Project entered Michigan State University's Academic Learning Program in September 1987 and graduated in June 1989. In this intervention, the teacher candidates were enrolled in a sequence of three nontraditional mathematics courses devoted to an exploration of numbers and number theory, geometry, and probability and statistics.[3] A methods course and a curriculum seminar drew on the content courses and field experiences to engage prospective teachers in reconsidering their notions about mathematics education.

Several assumptions guided our development and implementation of this intervention. First, the selection of mathematical content had to meet certain criteria: What does knowing this idea enable a student to do? To what other mathematical ideas is it connected? Does it require students to engage in *doing* mathematics—analyzing, abstracting, generalizing, inventing, proving, and applying? Second, the content and learning opportunities should require students to communicate their understanding in multiple ways: engaging in mathematical discourse among themselves and with the teachers using natural and symbolic language; writing about their reflections on teaching and learning mathematics; and using multiple representations—numeric, algebraic, graphic, geometric, spatial—to depict the mathematics embedded in problem situations.

The third assumption concerned our concept of a learning community. Our conception included a stable cohort of students who would engage in common study and experience over a two-

[3]See Lappan and Even (1989) for a fuller treatment of the mathematical content of the courses.

year period. Our image of community was richer than simply having groups of students work together on a problem and then report their findings. Our vision of community was a classroom where students and teacher together engaged in mathematical inquiry.

In this community, developing ways of knowing was a fundamental mathematical goal. This included ways of approaching a problem situation, ways of seeking additional information, ways of making a convincing argument, and ways of knowing that a solution makes sense. We wanted these students to experience individual, small-group, and large-group work within that community and consider what each can add to the development of mathematical ideas. We aimed to create an environment that fostered cooperative learning and teaching: a set of students working collectively to build mathematical schemas that they could use in flexible ways to approach new problem situations and a set of faculty planning, monitoring, instructing, and evaluating progress across courses.

Data Collection and Analysis

Data for the entire cohort of teacher candidates consist of field notes of all class sessions and video recordings of some, as well as audio recordings of small-group work. Questionnaires were administered at seven points in the study. We collected samples of student work that included written assignments and exams. In addition, we followed an intensive sample of four students. Data from our intensive sample include tape-recorded interviews conducted at eight points during the program, observations of their student teaching, and interviews with their mentor teachers and fieldwork instructors. In the third year of the study, we conducted periodic observations and interviews of our intensive sample in their first year of teaching to study both knowledge and contextual constraints in implementing a conceptual approach to elementary mathematics education.

To investigate the ways in which community was constructed, we analyzed classroom observation data for evidence of elements we take as constitutive of community: (a) teaching and learning is collaborative; (b) different approaches to problem situations are valued; (c) responsibility for understanding is shared; and (d) authority for knowing is internal and collective. To investigate the ways in which community contributed to changing preservice teachers' beliefs about what it means to know mathematics, how mathematics is learned, and the role of the teacher in the mathematics classroom, we analyzed classroom observation data together with student questionnaires, interviews, and written work.

Contributions of Community to Learning Mathematics

Establishing a Norm of Collaboration and Shared Responsibility for Understanding

The students who entered this preservice education program were quite unprepared for the experience they were to encounter in the mathematics classes. Andrea's[4] comment was representative of what every student we interviewed had to say:

> There is a difference between other math classes and this one. In other math classes you don't say anything. You just sit there and watch the professor write problems on the board all hour. In this class, you couldn't get away with just sitting there and [expect to] learn because you couldn't get anywhere. Right away I knew I had to change the way I thought about this and that wasn't easy at first.

Andrea made this observation during a class discussion about the nature of the problems the teacher posed. Posing "big problems" that did not lend themselves to direct, immediate, singular algorithmic solutions contributed to students' relying on each other for insights on how to tackle a problem situation.

But working together was not automatic. During the first course on number theory, we observed within the small groups a mix of collaborative investigation—students questioning each other, making suggestions about various strategies, trying to explain what they were doing and what they were getting—as well as individual attempts to solve problems. The efforts of Wanda, Chuck, Denise, and Lynn, four students who frequently worked together, are illustrative.

In the first course on number theory, the study of the structure of numbers was introduced with the "Locker Problem":

> In a high school there are 1000 students and 1000 lockers. The lockers are in a single row in a very long hallway. At the beginning of the school year the students perform the following ritual: The first student enters the building and opens every locker. The second student goes to every second locker and closes it. The third student goes to every third locker and changes the state of the door. In a similar manner, the fourth, fifth, sixth, . . . student changes the state of every fourth, fifth, sixth, . . . locker. After all 1000 students have passed down the hall, which lockers are open?

[4]Names of preservice teachers are pseudonyms.

A partial transcript shows the collective efforts of Wanda, Chuck, Denise, and Lynn at solving this problem. They decided to check out the first several lockers:

Wanda: So the ninth person goes to locker nine and opens it.

Chuck: What about the factors involved?

Denise: Seven stayed open until the seventh person got there. Five stayed open.

Wanda: These are primes.

Chuck: Four is closed.

Denise: But 4 isn't prime.

Chuck: So all primes stay open until that person changes the state: . . . So we know eventually all primes are closed except for one . . .

Chuck: One, four, and nine are open.

Denise: These are perfect squares.

Wanda: Let's try 4 squared.

Chuck: Just do 16.

Wanda: But you couldn't do just 16 because you might have multiples you have to close or open prior.

Wanda: [to the teacher who has approached this group] We're going to conjecture that perfect squares are open.

Teacher: Why? You have a very good conjecture but why? What is peculiar about square numbers? What is there about the structure of numbers so that primes are closed and composites are closed? [Teacher moves on to another group.]

Denise: Primes get touched only by that person.

Wanda: But why are square numbers open?

Denise: Well, the squares have two people passing over it. . . . Let's look at composite numbers.

Lynn: It [a composite] gets hit for each factor.

Chuck: Six is 2 and 3 but 4 is 2 and 2, and 9 is 3 and 3. Then why shouldn't composites be open as well?

Denise: How about if we go back to what you said: 4 is 2 times 2. When you go over it with the first two it closes, when you go over with the second two it opens. With nine, the first three opens it, the second three closes it.

The four pursued this problem together for nearly 30 minutes. It took another set of guiding question from the teacher to help them rethink what it would mean to have a repeated factor. Finally they concluded that only square numbers have an odd number of factors.

This same group worked much differently on the next "big problem" of the number course ("Magic Johnson and the Rookie"[5]). Tim had joined their group. In contrast to the collective efforts we observed with the "Locker Problem," this time Tim worked independently and Lynn and Denise worked together. Occasionally Wanda and Chuck would interrupt to see where the other three were with the problem. Wanda and Chuck seemed interested in what the others were finding, but they contributed little initially to helping find a solution to the problem. Only after Tim had made some progress in exploring the problem did they actively join in trying to understand what he had found.

Our observational data showed an increasing reliance on the collective efforts of members within small groups at problem solving over the three-course sequence. In a set of activities in the spatial visualization unit, students were to use cubes to construct buildings from sets of plans. When the teacher introduced the problems, Jason asked if a set of plans would produce a unique building. The teacher responded by telling students to keep Jason's question in mind as they constructed their buildings. Further, if they did build different ones, under what conditions would that be possible. Barbara, Andrea, Allison, and Kelly (a group that often sat and worked together) each began to create a building checking with each other as they went along. One set of plans produced several different buildings:

Barbara: Here's how you can add another cube to get a second building.

Allison: [pointing to a spot in her building] Here you can have one or two cubes.

Kelly: Here's my way. Let's see if it's the same as the others. Andrea's is different.

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[5]The problem: Magic Johnson has signed an NBA contract to be paid one-million dollars a year for the next 25 years. The rookie has agreed to be paid $1 the first year, $2 the second, $4 the third, $8 the fourth, and so forth. Who has the most at the end of 25 years?

That's four ways.

Andrea: I think there's even more! [She adds another cube to her building.]

Allison: Show me again.

Kelly: You could put one in front. [She adds a cube to Andrea's building and then checks it against the plans.] Oh no, that changes the front view.

At this point Barbara, Kelly, and Allison abandoned their buildings and moved around the table to look at Andrea's building. Andrea began to put cubes in various spots and they all checked each new configuration against the plans:

Andrea: [pointing out three places where cubes were hidden from view in the plans] You can have 1, 2, 1 in these spots or you can have 1, 2, and 2 . . . Ah! There's lots! You can have any of the options here.

Kelly: Summarize all that.

Andrea: What are the combinations? Three ways here combined with three ways here.

Kelly: How do we know we've got 'em all?

Andrea: Just look at the ones you can't change.

This group of four had begun the problem exploration individually. However, within two minutes they had abandoned their individual efforts in favor of a collective approach. What seemed to dictate the working relationships within groups at any one time was the nature of the mathematical task posed and the desire of the students. If the teacher set the task with an organization that needed to be changed as the work progressed, the students increasingly felt free to make a change.

The teachers' role in developing norms. Part of what contributed to establishing a norm of collaboration and shared responsibility for learning was the teacher's interaction with small groups—her probing questions, the ways in which she extended their thinking, her recognition that among the groups there were different levels of understanding. She acted as a guide, posing questions that allowed students to build abstractions and generalizations. For example, her second interaction with Wanda, Chuck, Denise, and Lynn on the "Locker Problem" helped them to look more closely at the structure of numbers:

Teacher: What distinguishes composite numbers from square numbers? Look at their structure and see if you can puzzle it out. . . . Try 36. Figure out who is going to touch 36. . . . Now try a number that is nonsquare.

The teacher's interactions with various groups exploring the "Magic Johnson" problem evidenced her awareness of different levels of understanding. In several groups she suggested they create a data table to record systematically the data they were generating. In a group that had already created a data table, she suggested they try to write some kind of mathematical model for the general case. Sometimes she asked questions to see if students had the problem conceptualized correctly:

Teacher: You've already figured out it has something to do with exponents. I'm not going to stay here until you get it sorted out but let's think about this. Not only do you need to be able to tell what the rookie earns in a particular year but you need to tell what the total is that he has earned up to that year—the cumulative earnings.

One group had an algebraic model and a student was working on a graphical representation. The teacher pressed the group to think about the various representations:

Teacher: Albert appears to be working on a graphical representation of the problem. Within your group ask what each of those representations adds to your understanding of the problem. Under what circumstances would you go to each one of those representations of the problem?

The role the teacher assumed in small groups was to ask important questions, guiding students in their struggle to make sense of the problems and their solutions. When she was satisfied that investigations within the groups had produced sufficient understandings, she brought the whole class together to discuss individual group efforts. In this context she again gently pushed them to consider weaknesses in their arguments so that more powerful and convincing generalizations could become part of the working knowledge of the community.

Valuing Different Approaches to Problem Situations

One of the goals of this intervention was to develop the mathematical power of prospective teachers. To do so entailed building a repertoire of strategies and representations that students could use to solve nonroutine problems. It meant helping students to become flexible, to see that a variety of methods might be applied to any particular problem situation. In the second and third mathematics courses, we observed an increased willingness on the part of the students to engage in mathematical investigations and an increased confidence in their ability to apply knowledge in unfamiliar problem contexts. Their exploration of the "Newsgirl Problem" in the final course is illustrative:

> The problem: A newsgirl delivers papers daily and once a week collects $5 from her customers. One customer offers her a deal. Each week she can draw two bills from a bag containing one $10 bill and five $1 bills. Should she take the offer?

Students were to analyze the problem theoretically and consider ways to simulate the situation. Small groups of students worked on the problem for about 25 minutes, and then the whole class discussed the various ways in which the problem had been analyzed and simulated.

Andrea, Albert, and Bart drew a probability tree in which the branches represented favorable and unfavorable outcomes. They reasoned that the probability of an unfavorable outcome ($2) was 2/3 and a favorable outcome ($11) 1/3. Theoretically in a three-week period, the newsgirl would get $2 twice and $11 once for an average of $5 per week. This led to a discussion among several students about the real-world situation and that a theoretical probability did not guarantee a particular outcome of an event. As one student described it, the newsgirl could go 10 weeks getting only $2. But another countered, "Yah, but she could also go 10 weeks and get $11 every week."

Tim's group figured the probability of getting $2 by recognizing the multiplicative nature of the event (5/6 chance of getting $1 on first draw, 4/5 chance of getting $1 on second draw: (5/6)(4/5) = 4/6). Alicia's group completed a probability tree showing every possible outcome. Jim's group used the idea of expected value. All groups concluded that the deal from the customer was a fair one.

The simulations suggested by the groups were as various as their analysis of the situation. Albert's group used the roll of a die for the first draw and pulled a chip for the second. Tim's group put slips of paper in a bag. Several students felt this was a good simulation because it was a model of the situation. Others said it was not particularly good unless you carried out many tries and that it would be hard to distribute paper slips randomly.

Chuck's group used six pennies, one of which was dated 1963 that they called the $10 bill. Lori's group used spinners, one divided into six parts (one $10, five $1), another divided into five parts (one $10, four $1). They reasoned that if the first spin was $1, then you would spin a second time. However, if the first spin was $10, a second spin would not be necessary. Several students talked about simulations that would be problematic, such as rolling a die or using one spinner. Their argument was that the situation required a model where they could remove one element.

During the entire group discussion the teacher participated only minimally, occasionally asking if any group had approached the problem in a way that was analytically different. She acknowledged every group effort as a legitimate way of approaching the problem, even though some were less elegant than others. For example, Alicia's group had used what we came to call "brute force and awkwardness"—listing all possible 30 outcomes in a probability tree. This was the first analytical tool the students had developed to investigate probabilistic situations, and some students persistently returned to it as a favorite mode of analysis, especially if the number of possible outcomes was a manageable size, as it was in this case. What was significant about this particular event, and many like it, was that students approached problems in various ways, offered multiple ways of investigating them, and argued the reasonableness of their conclusions.

Creating a total environment where teaching and learning are collaborative, where responsibility for understanding is shared, where different approaches to problem situations are valued can only take place over time.[6] The willingness of students to take risks in the whole group required a learning environment in which all students were treated with respect and where all ideas were taken seriously. As Sharon, one of the students put it,

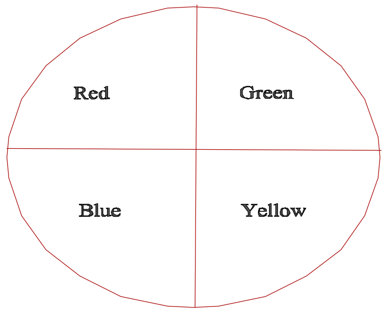
> She [the teacher] always used your answer, without any put-down, to contribute to class knowledge. Even if it was wrong, it might be a counterexample. You were never put down. If you said something, it was considered seriously. . . . What was a no-no was to not try at all.

The Shift in Epistemological Authority

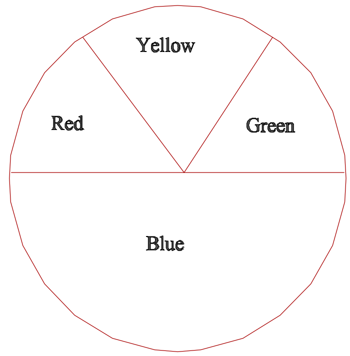
Perhaps the most significant development among the students was the shift away from the instructor as the sole source of authority for knowing. The students' exploration of the problem "Making Purple" is illustrative. "Making Purple" was assigned as a homework problem in the final course on probability and statistics:

> Given the two spinners below and two spins, which situation maximizes the probability of getting purple [red and blue]: two spins on Spinner 1, two spins on Spinner 2, or one spin on Spinners 1 and 2?

[6]In an earlier pilot of the 10-week number theory course, an openness and trust developed within small groups but it was never matched in the context of the whole group (see Schram, Wilcox, Lanier, and Lappan, 1988).

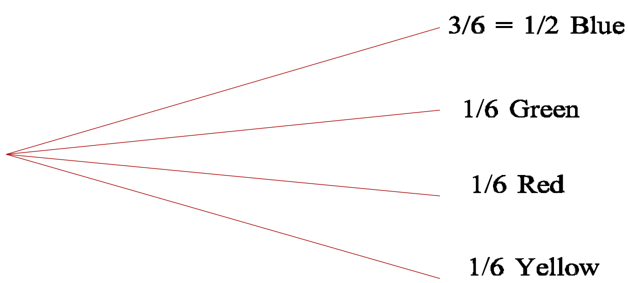


Spinner 1



Spinner 2

In the second class meeting following the assignment of the problem, the teacher began an exploration of the problem with the whole class. She began by asking whether students thought the situation was additive or multiplicative. Students posited that the situation was likely to involve multiplication because a complete trial required an action followed by another action. The discussion then moved to consideration of ways to analyze the problem beginning with the situation of two spins on Spinner 2. Allison suggested they draw a probability tree. Sharon volunteered to go to the board and draw the tree. This is what she drew:



Jason suggested that for some kids you might consider dividing the blue section of Spinner 2 into three parts so they could see it as three blues and then construct a tree with six branches where the probability of each branch was equal. Albert suggested labeling the branches as those that could lead to a favorable outcome (blue and red) and those that would lead to an unfavorable outcome (yellow and green).

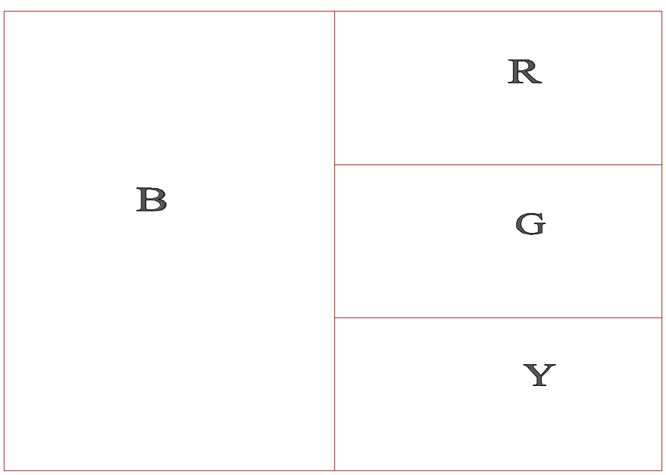
The discussion then moved to the second spin. They reasoned that the tree needed to be completed only for the red and blue branches since yellow and green could not yield purple. Barbara completed the tree:

Teacher: This is the stage of the game when you can run into trouble. Everything is in the tree but only if we understand what it represents. Tamara, can you reformulate the problem for us?

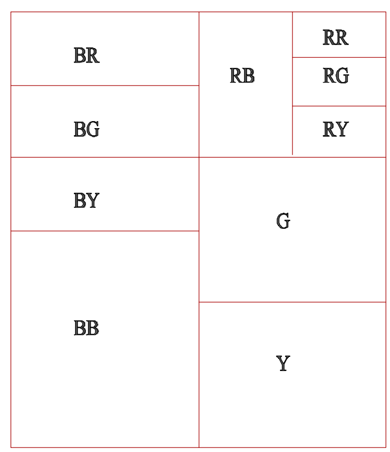
Tamara: We want to know when blue-red and red-blue are outcomes.

Teacher: How do we do this?

At this point several students offered arguments to support the multiplicative and additive aspects of the situation. Next, the class considered other models to represent the situation. Jim drew two models representing each spin on Spinner 2.



1st spin



2nd spin

There was considerable comment as Jim drew his model. "That is neat." "Wow, you could do this with water colors." "This is great for visual learners." "You could use colored transparencies and overlap them." When Jim had finished his explanation, the teacher asked if either model, the tree or the geometric representation, was a complete probability of the situation: "The responsibility for the whole class is to help Jim make the best possible, clearest model from the point of view of kids' learning."

Jim drew an elaborated model. Just as he finished there was a power outage and the room, which has no windows, was plunged into near darkness. However, the students continued the discussion with enthusiasm. Lori suggested using a coordinate grid to locate all the cells in the model. When the teacher noted that you could use two identical transparencies and physically turn one and place it on the other, Albert objected:

Albert: That's an arbitrary rule to rotate. How would you explain that?

Teacher: How did we make it make sense with the dice problem [an earlier problem with a similar structure]?

At this point students wheeled the chalkboard into the lobby and continued with their investigation. Albert went to the board, drew another version of the grid and then reasoned for himself, while the rest listened, that rotating it 90 degrees made mathematical sense.

This event was not an isolated incident. Consistently throughout the geometry and decision-making courses students evidenced a growing disposition to engage collectively in mathematical searches, applying multiple problem-solving strategies to unfamiliar problem situations. In the final course, the teacher paid explicit attention to having students consider a problem situation from the point of view of children's learning. What might be the different ways in which youngsters might approach a problem? What simulations, models, and representations might enhance children's understanding of the mathematics embedded in a problem and in what ways?

We observed among the students an increasing reliance on their collective ability to decide when a problem had reached resolution. In the number theory course, students tended to look to the instructor to tell them if their solutions were correct and if their ways of reasoning made sense. At the same time, the teacher resisted being drawn in by their demand to be told if they were right. She refused to be the authority who established truth. Instead, she insisted on questioning and collaborating with them rather than telling them. Over the three courses we observed a shift in the locus of epistemological authority—from a reliance on the teacher to their community of classmates and teacher together using mathematical tools and standards to decide about the reasonableness of processes and the results of investigations.

In "Making Purple," Albert challenged what he considered the "arbitrariness" of rotating the transparency. And he accepted the challenge put forth by the teacher to make sense of it for himself. Interestingly, the initial question—Which combination of spins maximizes the probability of getting purple?—was never answered. By exploring in depth two spins on Spinner 2, the class was comfortable that they could answer the question. They demonstrated they had the knowledge to analyze the situation, consider various ways to think about it, and draw on a repertoire of models to represent and make sense of the situation. At this point, the solution was straightforward.

Changing Preservice Teachers' Beliefs About Community

In the preceding discussion we provided vignettes of a classroom where a community of learners was constructed. Within that community over time we observed a cohort of students display a shared responsibility for engaging in mathematical searches, an increased confidence in their ability to apply knowledge in unfamiliar problem contexts, and a reliance on their collective competence to decide when a problem had reached resolution. Students themselves had become the judges of the validity of the arguments they put forward. We have considerable evidence of a change in their behavior as adult learners of mathematics. But was the intervention powerful enough to alter deeply held beliefs about how mathematics is learned or the use of small groups in the mathematics classroom?

Communities of Small Groups

There were two important sites in which a community was constructed. The first was in small groups. The cohort of 23 students in our project had been together in the program for 20 weeks when they entered the first mathematics course. In their earlier foundations courses, they had worked together in small subject matter interest groups—language arts, social studies, mathematics, and science—and had already formed friendships and working relationships, both in and out of class.[7]

All three mathematics courses were held in a room typically used for seminars and faculty colloquia. The room contained seven tables, arranged in a U-formation. Upholstered swivel chairs lined each side of the long rows of tables. From almost the first day, students arranged themselves at the tables in ways that seldom varied.

Initial groupings were naturally formed largely on the basis of familiarity. However, once group work became the norm, these grouping arrangements tended to solidify. Students found within their small groups others with whom they could work comfortably and where they could assume a role that fit with their sense of self. In a discussion in the mathematics methods course, the students talked about their perceptions of how they formed groups earlier. The notion of being comfortable in a group was echoed by many of the students. The students in one group were the least confident of all class members in their mathematical ability. They found support among each other. As Kim commented when asked why she chose that group: "I came in late and it was the group I felt comfortable with.

[7]Of the 23 students, 4 chose mathematics as their subject matter interest, 8 chose science, 6 social studies, and 5 language arts.

We had started working together outside of class winter term. It's easier to take risks in this group.

Two of the strongest personalities, Lori and Jim, were in a second group. They were confident in their mathematical abilities and often were able to see some of the subtleties in a problem situation before others. Lori said that on one occasion she had to sit with another group:

> I wouldn't feel as comfortable in one group as another. I would work with them but it would take me longer to do math with another group than my group. One day I came in late and sat with Anita, Karen, and those guys. It wasn't that I didn't work with them or didn't like them but it wasn't the same. It doesn't mean they didn't accept me or we wouldn't accept someone else but [pause] there is something there [about being comfortable].

Sharon almost always worked with Lori and Jim even though she was not nearly as confident as they. She explained how she found her way to that group:

> I just came in and sat with Lori and Jim and Linda and they included me even though they didn't know me and gave me work to do that I could do. And so I began to feel part of the group and they kind of dragged me along with them.

Kelly talked about a particular dynamic that she thought might account for choices in working groups. She was addressing Bonnie, a student with whom she frequently worked:

> You have a role with the group you're in. You sat with us and you talked and we listened. You were the leader; that is your role. Whether you were aware of it or not, you were the leader and comfortable with that. If you had been in the group with Lori and Jim, you might not have been able to be that leader. I keep thinking of the role you take in your group.

A few students worked with whomever they were near. Barbara had a hard time getting to class on time and sat wherever there was an available chair, usually on the right side of the room. Albert drifted about the room, sitting with different groups but often initially working alone on a problem before sharing his work with others.

The teacher chose not to meddle with the natural groupings that were established early on. The decision reflected her sensitivity to and respect for different personalities and levels of confidence with mathematics among the adult students. Although she thought there might be some benefit to reordering the groups occasionally, she worried that invading and disrupting their spaces could be counterproductive to creating a safe environment in which students would be willing to take mathematical risks.

We administered a questionnaire to the teacher candidates in our study on a number of occasions. Several of the items were intended to assess beliefs about the value of small-group work in the elementary mathematics classroom. One item posed the following question:

> In social studies classes students are frequently asked to work in small groups on assigned tasks. Is work in small groups appropriate for mathematics classes? Explain.

In their first response in Fall 1987 (prior to enrollment in the sequence of mathematics courses), nearly all the prospective teachers tended to think that group work might aid "slower" learners or help youngsters to review, as these examples of their explanations show:

> Yes, because people of higher skill are often able to explain math to peers in a way that might be [more] useful to them than what the teacher says.

> I feel that grouping in mathematics would be a good idea at specific times like maybe when students are reviewing material, studying theories or principles, they can help each other learn if problems arise. But, often the children should complete their own work first before they meet in the groups and then discuss the math subject at hand.

By the time of the last administration of the questionnaire in June 1989, they had come to value group work for very different reasons, as the following responses portray:

> Definitely. People who work together see the ways others think about things. The input of others is often helpful in forming your own views. It is also helpful to tell others how you are thinking. By verbalizing, you are more able to solidify your views.

> By talking about their understanding with each other, misconceptions can be detected. It is also beneficial when peers challenge each others' thinking (as well as the teacher).

Every student commented on at least one of these values they associated with small-group work: (a) communicating about mathematical ideas, (b) talking with others to clarify one's own understanding, (c) being more willing to take mathematical risks within small groups, (d) seeing the multiple ways in which diverse learners approach a problem situation, (e) learning how to work collaboratively, fostering cooperation and development of social skills, and (f) developing independence as learners. Their reflections in the final written assignment add to this picture:

> As students we worked independent of Glenda, relying on her only to guide the problem solving situation that we were engaged in. We were organized into groups

> decided upon by ourselves. The students summarized (with the occasional help of Glenda) what we discovered. We felt socially responsible to ask and answer questions posed by students as part of the class. (Lori)
>
> The expectations in this classroom are higher than the average traditional room, for we are not simply supposed to "do" a problem, we are to: understand it, communicate our understanding, see the connection to other concepts, know different ways to problem solve a situation and generalize a formula. (Lynn)

Several of the students we interviewed commented on the value, if not necessity, of working with others:

> Andrea: You had to learn from each other. . . . A lot of times I look at a problem, it's a strange problem situation and you don't know what to do. And, you start passing ideas around and trying to think of some way to approach the problem and pretty soon, one person's like scratching around and then you catch on to what they were doing and do a couple of scratches yourself and pretty soon you put everything together.
>
> Lori: When I could see someone else struggle over a problem and come to the same understanding, come to the place where I was, that helped me to learn more. And I think it helped them to learn more.
>
> Allison: When I could explain a problem to someone else and help them, then I knew I was successful. If I could verbalize instead of just applying. It's like sometimes you feel like you are almost right there and then when you talk it out you see what you don't get. Or you figure out that you really do have an understanding.

Community Within the Whole Class

The second site in which a community of learners was constructed was within the whole class. Whole-class discussions primarily served three purposes: (a) posing problem situations, (b) offering conjectures and arguments about problems and their solutions, and (c) reflecting upon understandings and the connections and relationships among various mathematical ideas. The teacher frequently began the class period by posing a problem situation for students to grapple with. These problems did not lend themselves to obvious algorithmic solutions. Often students explored the problem in small groups but there were occasions in which the investigations occurred in the whole group or individually.

Regardless of the mode of exploration, there were opportunities to share results of group and individual inquiry in the context of the whole class. During interviews students often talked about the value of whole class discussions:

Anita: She [the teacher] doesn't start off by simply explaining how to do the problem so we can check our answers. Instead, various volunteers present their thinking strategies and approach to the problem. . . . She explores each strategy with us. . . . She asks questions to the class and the person presenting to help them further their thinking and to help with clarifications.

Andrea: Well, it's really interactional, like we have a set of problems to work on together and it makes it a lot easier because you hear someone's idea and you have your own idea and then pretty soon you end up arguing and working things out together. . . . Talking about mathematics enables you to see somebody's reasoning.

As a final assignment, students were asked to select and analyze a typical class from the intervention. Many included comments about whole-class discussions. The following excerpts from their writing represent this well:

> Discussions were clearly non-traditional in that teacher and students played equal roles in participation, initiation, and questioning. (Anita)

> During every lesson we always had a question to start off with to think about and investigate within our groups. . . . We then came back to a whole class again and compared our ideas with others in the class. . . . Our responses are made in more than one mode and the linkages are then made to enrich the ideas that we independently investigated in groups. (Amanda)

> This is the time we usually try to formulate a general rule to use to find "n" during a given condition. (Lynn)

Responses to class assignments are not sufficient evidence that student beliefs about the value of small- and whole-group work have been altered. In fact, one might worry that students give teachers what students think they want to hear in such assignments, knowing their grade may depend on how they respond. But by triangulating data gathered through classroom observations, questionnaires, and interviews, we feel confident that the intervention contributed to changing these students' beliefs about the value of group work in their experiences as learners of mathematics.

Implications for Creating Community in Their Own Classrooms

Initial Efforts in Creating Classroom Communities

Because the learning of mathematics was embedded in a context of learning to teach, developing subject matter knowledge could be linked to developing pedagogical content knowledge. Reflections on differences within the community of the teacher candidates themselves—how they learned, what they focused on, the questions they asked, the strategies they favored—helped them to appreciate divergent views in the classroom and to talk about children's learning in more complex ways. They talked about group work, nonroutine problem situations and multiple representations as powerful ways to explore mathematics and construct mathematical knowledge.

We wanted to follow a group of our students in student teaching and first-year teaching to see in what ways and to what extent they were disposed and able to create a community of learners within their own classrooms as they taught mathematics. In this section we provide sketches of the efforts of two students in our intensive sample as they worked with children.

Linda. In her student teaching, Linda consistently tried to create opportunities for children to talk with each other about mathematics and make sense of mathematical ideas for themselves. She developed a unit on fractions for her fourth graders. On one occasion she used the daily "lunch count" as a problem situation. First she asked the children to represent the fraction of students who were present in class. Some students wrote 25/29, others wrote 29/25. She raised this with the class: "Here's a couple of different things I see people writing down. Could someone explain what this [25/29] means? . . . Could someone explain what this [29/25] means? . . . Which one of these describes the situation we have?"

Together, the class determined which fraction was appropriate. Then she posed the question of what fraction of the children would be getting hot lunch. At this point, controversy arose. The children debated whether the "whole" was the number of students enrolled in the class or the number present this day. After much back-and-forth, the class reasoned that the "whole" should be the number present because those who were absent would not be having lunch at school.

In this context, Linda had a great deal of support from both her mentor teacher and her fieldwork supervisor. Her mentor was particularly interested in these "new approaches," although she did not have experience or knowledge to give Linda much help. What she did provide was the space for Linda to develop problem-solving situations out of the daily experiences in the classroom even if it might mean taking "extra" time. When Linda did run into trouble, it was her lack of subject matter knowledge that was the constraint. For example, she did not understand the distinction between using fractions to represent parts of a whole and parts of a set. She simply saw one as a continuation of the other, as the next lesson in the text. When she introduced these notions to the children, there was considerable confusion. In an attempt to help students better understand the idea of parts of a set, Linda kept going back to parts of a whole, a strategy that only led to further confusion, for her as well as the children. Her field instructor described the situation this way:

> An analogy to a road map helps me think about what is missing for Linda. She knows that a big picture exists. The big ideas can be represented by cities. But some of the roads connecting the cities seem incomplete. She doesn't always understand the subtleties. For example, in her fraction unit, she got into trouble when she introduced *her* representation of equivalent fractions. She didn't understand the big conceptual leap it required for kids.

Linda is now teaching in a private school. She and a colleague work with two groups of students, one composed of first, second, and third graders, the other, fourth and fifth graders. At the beginning of the school year, students were scheduled to meet for group math instruction once a week. The remainder of the time for mathematics was spent with children working independently. Linda's own experiences in our study, as a learner and student teacher of mathematics, convinced her of the value of students working together as members of a community. Committed to this view of schooling, she negotiated with her colleague a schedule of two to three days a week in which small-group and large-group learning experiences might be incorporated into the mathematics program.

Even with the principal's support for these changes, Linda has encountered resistance from some staff and parents. Those who are resistant do not see a need for change from current arrangements to do things differently. Their concern is that the kinds of mathematical activities Linda wants to engage children in will take too much time, thereby limiting the amount of material she can cover. It is not clear if Linda will be able to withstand the pressures to conform. But she has demonstrated a disposition to create a mathematics classroom where members of a community of students and teacher together are "mathematical meaning makers," and the community *acts* on that conviction.

Allison. Allison exemplifies the beginning teacher who constantly struggles with the tension of wanting to teach in nontraditional ways in the face of what she perceives to be overwhelming contextual constraints. In her student teaching, Allison used some exemplary curriculum materials aimed at developing students' conceptual understanding of perimeter, area, surface area, and volume. She grouped her middle school students for activities but then did not capitalize on the materials or the grouping arrangements for their power to engage students in inquiry. She spent considerable time at the overhead, providing examples, asking questions, calling on individual students to answer, and writing down their correct responses.

She constantly worried that the class period was too short and that there was too much material in the curriculum that her mentor expected her to cover. What she ended up cutting out were the explorations that would allow students to create meaning for mathematical ideas. For example, one day she spent nearly the entire period writing formulas at the overhead, plugging in numbers and doing the calculations, and having students copy this in their notebooks. At the end of the class she told us, "I get so frustrated. These classes are so short. I don't have time for the discovery mode. I feel like sometimes I just have to *tell* them, you know, tell them the formulas. It's so frustrating." In the final term, Allison returned from student teaching dissatisfied with her attempts to create a classroom where youngsters were encouraged to work together to make sense of mathematical situations. This concern focused her thinking in the final mathematics course and the curriculum seminar.

Allison is currently teaching fourth graders in a small rural district. As part of her job interview in August, she had to teach a group of fourth and fifth graders in the presence of several principals from the district. In preparation for her interview, she called us for some feedback on what she was planning. She had some good ideas and some interesting activities, but she was not focused on the mathematics or what students might gain from doing them. We pressed her to focus on the mathematical idea and then consider what activities and representations would help youngsters to develop an understanding of the idea. Her final plan incorporated small-group work, the children coming back together as a whole group, sharing patterns they had discovered, making predictions about the continuation of the patterns, and creating ways to test their predictions. She had many good ideas, but she needed help to push her thinking beyond just interesting activities.

Classroom observations and interviews in her own classroom indicate that she seems less concerned about providing opportunities for her fourth graders to engage in mathematical investigations. She continues to struggle with some of the same constraints that she encountered in student teaching. Time continues to be a factor, although here it is more a matter of the time required to plan and locate or create materials. But there are additional constraints. She has been told by the principal and the fifth-grade teachers that they expect the students who leave her class to have mastered computational facts. To that end, she has students spend considerable time working individually on drill-and-practice and timed tests.

She feels overwhelmed by the amount of preparation required to plan and teach many subjects. On one occasion when we observed small-group work, children together created some interesting problems related to whole number operations. But in an interview following the lesson, Allison seemed more concerned about what she perceives to be a wide range of mathematical ability among her students. In late winter, she implemented a self-paced, self-testing mathematics program that the fifth-grade teacher recommended as a way to deal with perceived differences. Each student works individually on a set of computation exercises, checks with the answer book upon completion, and moves on to the next set of exercises.

During one observation, Allison had the youngsters working on an ecology unit. On this day they were given data on the per capita waste generated and recovered by a dozen industrialized countries. Students were given the task of computing with a calculator the percentage of waste recovered by each country. The youngsters diligently carried out their calculations, recorded their answers in the blank column on their data sheet, and answered some questions about various countries' efforts to recycle waste. In a conversation following the lesson, we asked Allison if she intended to do anything further with this lesson. When she indicated no, we suggested she consider having the youngsters make graphs as another representation of the data on waste generation and recovery.

When we returned for a final observation the next week, there were a number of bar graphs on the bulletin board created by the children using the data from the earlier lesson. What was particularly interesting was the variety of ways that youngsters had chosen to represent their data. Some had displayed single comparisons of waste or recovery. Others had combined these features to make rather elaborate graphs. Allison and the students seemed proud of their products. Allison was particularly appreciative for the suggestion and how well her students had done on the task.

Allison is not reluctant to ask for help from those around her. But at present, it seems doubtful that she has colleagues who can help her to think about how to create a classroom where learners engage collectively in mathematical inquiry. Considering the workshops her principal has had her attend and the kinds of suggestions she has received from colleagues, two issues seem to be of concern: how to manage the classroom efficiently and effectively and how to ensure the computational proficiency of diverse learners.

The Remaining Challenge

In this paper we have examined an intervention in an elementary teacher preparation program designed to develop in teacher candidates a conceptual understanding of mathematics and a conceptual approach to mathematics education. Our analysis suggests that the intervention produced significant changes in prospective teachers' beliefs about themselves as learners of mathematics, what it means to know mathematics, and how mathematics is learned. We believe our data support the assertion that creating a community of learners engaged in the *doing* of mathematics can be a powerful influence in increasing teacher candidates' self-confidence as mathematical problem solvers. We also believe that creating a community of scholars takes place over time and requires creating a total environment where students will take risks to make conjectures, offer arguments in support of assertions, and assume the authority for deciding about the reasonableness of mathematical representations and solutions.

Our students' efforts at creating classrooms of their own that embody a community of students engaged in mathematical inquiry has proved far more difficult. What this study has uncovered is that beginning teachers who are committed to creating a different environment in their mathematics classrooms need the support of others who share their vision. This raises questions for all mathematics educators who are attempting to transform mathematics teaching and learning in our elementary schools by reforming teacher education programs.

How are we to overcome the perceived contextual constraints that lead beginning teachers to fall back on more familiar and traditional practices once they have left the university for their own classrooms? What kind of support is needed in the induction years for teachers who would institute practices that are likely to be questioned in traditional school settings? What responsibility do teacher educators have for providing some of this support? Can we extend the notion of community beyond the preservice program? What kinds of communities would need to be created among professionals in schools and how can we equip our students to be advocates of such communities? These questions deserve our serious and continued study and our best efforts at creative solutions.

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