

## ASSESSING ASSESSMENT: INVESTIGATING A MATHEMATICS PERFORMANCE ASSESSMENT

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I have taught high school mathematics for 14 years. Over the past several years, I have become concerned with my students' understanding of algebra concepts and skills. Numerous research findings point to the lack of mathematical problem-solving skills and conceptual understandings on the part of our nation's adolescents (National Research Council, 1989, and National Council of Teachers of Mathematics (NCTM), 1989). After my lectures and whole-class discussions of the concepts, my students could perform the computations quite well, but seemed to have very little understanding of what they did and why. When I asked them to explain their reasoning, many seemed unable to go beyond simply telling what they had done to get the answer.

To alter this situation, I have tried several different strategies to enhance students' understanding. For example: To help students discuss and share ideas, I used small-group cooperative learning, oral group presentations of topics and problems, and written assignments about mathematics. My assignments seemed to help students by requiring them to reason about mathematics and justify claims and responses. Discussions in their small groups increased in length, frequency, and quality. I heard students say to each other, "That's fine, but why?" or "We need to put this into words the rest of the class will be able to understand."

With all these changes in strategies, I began to wonder about whether my traditional tests were sufficient for assessing student understanding. In trying to think about assessing student understanding differently I began to focus my attention on what the *Professional Standards for Teaching Mathematics* (NCTM, 1991) says about assessment. An important issue raised by the *Standards* is to "align assessment methods with what is taught and how it is taught" (p. 110). Faced with the problem of assessing what I speculated to be a new kind of mathematical understanding for my students, I began altering my tests both in content and form to find out what my students did and did not understand. I wanted less computation required on the tests and more written explanations about solutions to problems. I have tried group assessments in the form of presentations of problems or topics to the class as a whole. I assessed individuals based on their contribution to the group presentation. I also tried giving the groups a problem to solve and having them write about their solution.

All of these changes provided me more information (about my students as a group and as

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<sup>1</sup>The author teaches mathematics at Holt High School. He is very grateful to Michelle Parker and Pam Geist for their hours of editing and encouragement.

individuals) than simple computational tests. However, the changes left me feeling that I was not getting an accurate assessment of what my students *understood*. I kept wondering if my students understood more than I was able to give them credit for, or perhaps less. Furthermore, how could I "see" any lack of understanding in a way that would help both the student and me identify it and talk about it? My concerns were echoed in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), which states, "It is not enough for students to write the answer to an exercise or even to 'show all their steps.' It is equally important that students be able to describe how they reached an answer" (p. 140).

My concern for trying to understand what my students are learning in Algebra 2 led me to design their final exam for the 1990-91 school year as a "performance assessment." Performance assessment can mean a variety of things. The California Mathematics Council (1989) in *Assessment Alternatives in Mathematics* provides alternative ways to think about how to assess student performance. The idea of performance assessment provided an alternative to a traditional exam in which students solve a variety of problems, individually, using paper and pencil within an allotted exam time. The performance assessment offered students the opportunity to discuss a limited number of problems representing a wide range of concepts and to solve them in cooperative learning groups prior to the exam. One of the primary goals in this assessment was to have students be able to justify how they solved the problems. During the first half of the exam period, four panels of adult "judges" each listened to a cooperative group discuss their problem solutions.

While half the groups were doing their performance assessment, in my classroom other cooperative groups wrote about solutions to three problems similar in style to those described above but differing in content. I asked these students to construct their solutions with as much detail as possible, though they did not need to do the actual computations for the problems unless they felt it lent validity to their methods. In the second half of the exam period, the groups changed places.

### **Background**

Holt High School is a small Class-A school accommodating grades 10 to 12. Located in a blue collar and middle class suburb of Lansing, Michigan, our student population is predominantly white. A 1984 survey of high school graduates shows that about 78 percent of the school's students take some college courses. Students are required to take two mathematics courses in grades 9 to 12 in our district, approximately 60 percent take Algebra 2.

Algebra 2 is a yearlong course in which students attend 60 minutes of class per day. In my Algebra 2 course last year, I had 28 students with a wide range of mathematical abilities. I assigned students heterogeneously to small groups in the beginning of the year based on their Algebra 1 and

Geometry scores and their reading scores on a nationally normed achievement test.

After I assigned students to groups, I made changes only when I saw that a wider range of knowledge and expertise would help the students better understand the mathematics. Students were not allowed to change groups for social reasons. If a group had a social problem, a counselor,<sup>2</sup> who worked regularly with the class, helped resolve conflicts.

### **Preparing for the Exam**

Three days prior to the exam, I gave my students six problems which drew on the main topics discussed over the year in class. I also gave them an explanation of how the final would be conducted, which read as follows:

You should find six problems included in this packet. You should work on these problems as a group as well as on your own time. During the exam period you will be asked to explain your results before a panel of judges. Each member of the group will be able to explain each problem by themselves. Other members will be present but will not be able to offer ideas on an individual's problem. You will not know which problem you will be asked so be sure to study all the problems. In your explanations include samples of graphs you may have used, calculations you may have done, charts you made up and any other information you feel will help the judges understand what you know. Do not write a script that you intend to read as this would only prove you can read.

Each problem included some computation along with opportunities to explain and make judgments based on the results of the computations. In these problems the students were either given a situation in which they had to derive data for the problems or they were given a set of data in which they had to decide how to analyze and expand the given information. Creating problems that would provide opportunities to show mathematical reasoning in multiple ways, while also being interesting and worthwhile, proved to be a major task. I worked to design the problems for several weeks along with asking anyone I could find for ideas. With the help of many of my colleagues I was able to write suitable problems.

I include three problems here illustrative of the range of questions students faced.

1. You and your partner have decided to go looking for a buried treasure described on a scrap of paper found in the basement of an old house. The only clues to the treasure's location is

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<sup>2</sup>The school counselor, Jan Wilson, and I participate in a classroom research project about high school mathematics students' self-esteem and sense of mathematics efficacy.

the following:

"The treasure is buried in a spot that is the same distance from the boulder as it is from the railroad tracks. It is also . . ."

And the rest of the information is missing. But some other clues you may be wise to consider are:

- 1) the distance from the track to the boulder is 11 yards.
- 2) consider the tracks as the directrix.
- 3) keep all of the units in yards or feet.

Keep in mind the distance of the treasure from the railroad track is interpreted as being the length of the perpendicular drawn to the tracks from the treasure.

2. Hooke's law states that the force  $F$  (weight) required to stretch a spring  $x$  units beyond its natural length is directly proportional to  $x$ .

You have a spring hanging from the ceiling in the classroom whose hook is 8 ft. above the floor and you want to stretch it down to 3 ft. above the floor in order to hook it to Mr. Lehman's belt loop. Devise a plan to determine how much weight would be needed to pull the spring down. What would you need to consider? Be as complete in your strategy as possible (What steps would be needed?). How would you know if the spring would lift Mr. Lehman or not? When wouldn't it lift him at all?

Create a set of data to prove your conjecture.

3. Suppose you are a doctor doing research on cancer cells. You have found a certain type of cancer cells are growing as follows:

Weeks	0	1	2	3	
Number of Cells		1	4	16	64

You experiment with different drugs and EUREKA! XI3V causes the cancer cells to stop all further growth and the cells start disappearing at a rate of 10,000,000,000 per hour with a maximum of 5 doses per day. More than 5 doses per day will destroy the patient's liver and kidneys and the person will die.

If you have a patient with this type of cancer and you have estimated that they have about  $2.814749767 \times 10^{14}$  cancer cells, how long have they had the cell growth occurring?

How long would you prescribe your patient take XI3V in order to make sure that all the cancer cells disappear? How many doses will this take? Give your answer in a reasonable

unit of time (i.e., days, weeks, months, or years; which ever seems to be the most useful).

If another patient comes in who has had this cancer for a year and is only given an estimated 5 years to live unless you can get the number of cells in her system below 10,000,000 within the 5 years so her body can start to repair the damage the cancer has done, would you put her on this medication and give her hope for a continued life? Be very clear in you explanation and have the appropriate figures to backup your determination.

The students had three class days and one weekend to work in their peer groups on solving the problems and constructing explanations that provided support for the solutions. When I first passed out the problems, I expected that students would waste a lot of time in the beginning and get themselves into a time bind towards the end. Yet I was pleasantly surprised; both the observers<sup>3</sup> and I noticed that students used their time very well during the three days. We also noticed that the conversations went beyond simple computation to talk about why different individuals solved the problems in certain ways and what alternative approaches were possible. The students worked very hard but did not seem to panic or be under the tremendous pressure I was used to seeing in traditional reviews for finals. The class seemed to have an atmosphere of seriousness, but also of confidence. Students seemed to believe they could solve these problems in ways they could discuss.

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<sup>3</sup>The observers included Pam Geist, a doctoral student studying mathematics education at Michigan State University, and Jan Wilson.

## **The Judges**

I organized judges into panels of three persons that included one person who knew the mathematics subject matter necessary to solve the problem, one person who was not as strong in mathematical knowledge, and one prospective secondary mathematics teacher studying teacher preparation at Michigan State University. The mathematics person could focus on the computation and the logic of students' explanations and mathematical understanding of concepts. The non-mathematics person could focus on the confidence level of the student. As I explained to my students, those judges should have enough confidence after hearing the student's explanation to entrust the student to solve problems for them. I asked prospective teachers, scheduled to student teach in the fall, not only to help in the assessment of the computational algorithms used to solve the problems but also to judge the mathematical conceptual understanding students had. I also wanted the prospective teachers to learn through experience that, even though a teacher may use many strategies to support and develop student's learning and understanding, students may still not conceptually understand some content. I kept myself off the judging teams since I believed I would bring a set of preconceptions about students' abilities and understandings and thereby constrain what I could really "see" students doing and thinking.

Before the actual assessment I gave the judges guidelines and evaluation forms for their task (see Appendix). I developed the criteria used on the evaluation forms from similar evaluations I had used during the school year for individual student and group presentations. I believed students would feel comfortable with these evaluation categories since they had seen them during the year. Furthermore, these six categories defined what I was trying to assess about my students' mathematical knowledge without being too burdensome to the judges. I also wanted a form that would not get in the way of students' and judges' discussions. I gave each student a copy of the evaluation form when I passed out the exam questions so they would know what they were being judged on as they were preparing.

I allowed the judges to choose whether they wanted to use the evaluation form I provided or simply give me written comments with a numerical score within the range of points allowed on the form. Most judges found the form very usable and stuck with it, just adding comments to the bottom. A few decided they could better reflect the students' understanding by using more extensive written comments. Both methods seem to work quite well as far as helping me know what my students understood. I found I had no problem evaluating students with the combination of methods used by the judges.

I created two main categories on the form, "Mathematics" and "Presentation," since I wanted to clarify for the students what they were being judged on. The "Mathematics" section reflected what I

believe are the four essential characteristics used in solving mathematical problems. These were also the components we as a class had discussed and focused on throughout the year.

I included the "Presentation" section since I wanted the students to know that just doing the mathematics was not enough. They would have to communicate their understanding of the mathematics to the judges. Although I did not want to overemphasize this component (presentation is not more important than mathematical understanding), I did want it to be a part of the process.

I asked the judges to keep in mind my goal for the final exam, which was to understand what my students understood. Also, I reminded them that this was the first time most students had faced an assessment situation like this and, therefore, the students might be nervous. I cautioned judges about using leading questions that might cause students to explain problems without truly understanding them. Yet, I encouraged judges to use probing questions to help students, when necessary, get started on their responses. I wanted students to be able to say something and feel confident that their preparation for this final benefitted them. If students were able to provide reasonable explanations during their discussions, and not just get hints from judges that would make it easy, I could be somewhat assured that students' explanations reflected their understandings.

### **The Day of the Exam**

I reserved the library for the entire exam period (which was, under our school policy, an hour-and-a-half long). I arranged the furniture into four areas so that four groups could be taking the exam at the same time. Each group had as much privacy as possible during the exam. I circulated around the room in order to handle procedural questions if they arose.

During the actual assessment, students went before the panel of judges as a group, but only one student presented a problem. The judges picked which problem each student presented, therefore requiring every student to be able to discuss all the problems and not just the one or two they felt most comfortable with. After the judges heard a student discuss a problem, they would open the floor to other students in the group who wanted to add anything or refute what they had heard. I set it up this way because I wanted to know what each student understood but at the same time I did not want the students to feel totally alone (without the peers they'd studied with) before the judges.

I combined the information I received from the judges and the problems the students did while in my classroom for a final exam score. I used the average of the three judges' scores for two thirds of the total exam score; and the in-class problems counted for one third. These scores were combined to make up 20 percent of the students' semester grade.

## What I Learned About What My Students Learned

The results of this assessment gave me plenty of information to digest about my students, my teaching, and our curriculum. First, I learned that many of my students were still only superficially learning and understanding the mathematics. In their groups during the year I overheard excellent discussions about issues and topics we studied in mathematics, and they were also getting better at writing explanations and justifications. However, when it came to explaining the mathematics to the judges, they were only able to tell the steps they took in solving the problem. They fell short when it came to explaining why they approached a particular problems in a certain way. A common response judges heard was, "That's the way we did it in class."

I am still trying to figure out why what we did in class did not translate into better performances during the final. I suspect that some of the students froze in the testing situation despite all I had done to help them relax. Some of the students seemed to have trouble explaining problems individually. They did not have their partners to provide them with some connecting ideas that would allow them to give a coherent explanation of their understanding. Finally, I wonder if the students needed more practice throughout the school year with performance assessments since the practice might help them to understand better what they need to do to prepare and carry out a good discussion of a mathematical problem.

In addition, I was surprised that some of the students I expected to do well didn't, and some I *didn't* expect to do well did! I think some prompting from judges helped these students begin to respond to the questions. While several students did well on their own, others gave good explanations after the judges asked several questions to help them focus their thinking. I felt especially pleased about this finding. If these students were taking a traditional final examination and got stuck on a problem, they would probably be doomed to be unsuccessful and probably continue not understanding.

On a performance assessment I could tap into what they actually understood. I also could sort out their misunderstandings from simple computational mistakes common on traditional final exams. The performance judges could help students, through probing, sort out conceptual ideas from mistakes based on incorrect computations.

Another way this exam differed from traditional finals is that I gained information from three different professionals about each of my students. Each judge helped me piece together a picture of my students' understanding that would not have been possible on a typical final. The judges' comments reflected a wide range of observations about what mathematical understanding my students had. Most judges focused on what sense students could make of the mathematics they were doing. Typical of these kinds of comments was this example: "The student was accurate mathematically in solving but did not manifest very deep understanding of what the problem was about." Said another judge,



"Started by stating the sense of the problem—the relationship between pressure and volume."

Often judges commented on how explanations and calculations fit together with the problem the student was solving and his/her understanding of it. Here are three comments about one student:

He calculated the correct equation for the parabola. The only thing he was unable to do was explain the formula [distance] he used to get his equation for the parabola. Other than that his explanations were very good.

Did not understand derivation of formula for parabola—could not provide explanation for why formula works—however set up problem nicely, clearly understood problem.

[This judge addressed the comments to the student] I hope you continue with your agility and explanation based on the graphical representation of this problem. That's important. Push yourself on why the formula/equation works.

The judges seemed to agree that this student could perform the calculations correctly. Yet they all pointed to the student's weakness in being able to explain how or why the formula worked. He seemed to be unable to make the connections as to why he would use the distance formula, though he knew it was necessary to solve the problem. As a teacher I learned that this student knew how and when to use the formula, but could not say why it worked—which is what I want my students to be able to do. What I learned about this student I might not have learned on a traditional final exam.

Another set of comments provided another picture:

Explained that she is just doing the problems like the book said. [She didn't know why she "logged" things to solve for  $x$ ]. Did not really explain why she did things very well. However, she was able to interpret her results and seemed to understand what they meant.

I asked [student] what a log is, and she said "some number to a power" and could explain nothing more about the concept. Throughout her performance she also kept saying she didn't know if she was "right". All of these comments point to an emphasis on procedures—which for the most part were passable until #3 where she divided instead of multiplied [even after a judge gave her strong prompts].

[Student] has an attitude problem. She thinks that she really understands more than the other people in her group and she may be partially right but she has a long way to go. When questioned she seems to think that it doesn't matter if she's wrong if it is her opinion. She doesn't seem to realize that in math everything isn't wide open, that there are more than opinions. She worked on problem four and immediately identified logs. She said she doesn't really know what a log is or what it means to log both sides. Most

people don't know. She did most of the problem well and was articulate. I couldn't judge her accuracy not having done the problem. One serious error was finding that the treatment would take 1,172 days and dividing by 5 to find the number of doses. When questioned she didn't revise and simply said she might be wrong. A judge asked about 10 days and how many doses that would be. She seemed to understand that 2 was unreasonable but didn't want to think about it at this point. She did do a good job explaining why it makes sense that it would take longer to get rid of the disease than it would take for disease to grow.

These comments give me a lot of information about this student. In class she always offered suggestions and usually could derive a correct answer. Her classroom participation led me to think she understood the concepts very well. However, the judges' comments allow me to see that this student is very capable of doing the computation without having the depth of understanding I had hoped for. On a traditional test she would have made the one mistake with the division for which I would have taken off a few points thinking she had just made a simple mistake. I would never have suspected the depth of her misconception would go to the point where, when confronted with it, she would choose to stay with it even though she would admit it was unreasonable. If I had this information earlier in the year, I would have been able to address some of these misconceptions to work towards further understanding.

From this student's comments I learned that she was able to perform the mathematics and understand most of the concepts in the problem. However, with some of the information she chose to use, she did not understand where it came from. In a traditional paper and pencil exam, with a few lines of computation to illustrate what a student knows, I might have never known that the student did not really understand the distance formula.

While learning about my students' substantive mathematical understanding, I also learned about their affective mathematical views. I learned that students seemed to enjoy this type of assessment. They felt confident they could do it! Afterwards, several students told me that they felt good about the exam and enjoyed taking the final this way instead of working problems for one-and-a-half hours. They felt they demonstrated what they really knew. These responses gave me information usable in answering another question I have wondered about: How can we help our students feel good about themselves in relationship to mathematics? During the past two years I have been working with Jan Wilson in trying to find ways to help our students improve their self-efficacy in mathematics. As stated in *Everybody Counts*, "In the long run, it is not the memorizations of mathematical skills that is particularly important—without constant use, skills fade rapidly—but the confidence that one knows how to find and use mathematical tools whenever they become necessary" (National Research Council, 1989, p. 60). When we talk about student self-efficacy, it is this level of confidence that we are referring to.

Since only some students volunteered their comments, I cannot generalize about all students in the class; perhaps there were several students who did not like the exam but who chose not to tell me. However, I have been teaching long enough to know that if students truly dislike something, they usually let you know one way or another. Also, by looking at the expressions on students' faces during and especially after the final was done, I was able to get a sense of how they felt about it. I did not see the strained and dejected looks I usually see during and after finals. Rather, I saw students who felt they had accomplished something. They were congratulating each other with "high fives" and commenting on how they felt they did. They also offered alternative ways of explaining a problem to their peers that the presenter had not used before the judges.

Several students who normally did not do well on exams were pleased with their presentations. One student commented that he was grateful for the judges taking the time to ask questions since he knew the information but had trouble finding the right words to describe it. This was similar to what the judges said about him. For this student to walk away feeling good about himself in relationship to a mathematics assessment was worth all my efforts to plan and organize it. The next day when he found out he got a "B" on the exam he literally jumped two feet off the ground and went down the hall screaming to his friends.

### **What Now?**

As I enter the 91-92 school year, my task is to use what I learned from this type of assessment as I plan for instruction that will continue to improve my students' conceptual understanding of the mathematics. I now know that good conversations in class do not always transfer into good understanding down the road. I must look for ways to help students transform in-class conversations into meaningful understanding. Transforming what I learned into realistic changes in my classroom is the hard job that lies ahead.

I also have to design a method of doing performance assessment throughout the year. I cannot wait until the end of the year to gather this information since I can better help students' understanding through ongoing assessments. Having four or five performance exams during the school year could help me gain an understanding of my students and allow me to help them be reflective of their growth and change. This would provide me with better checks on their understanding, and how their sense making does and does not fit with my instruction.

I am challenged by some hard questions about my instruction, the curriculum, and general conditions of learning high school mathematics. First, how does a teacher come up with problems that lend themselves to a performance type of assessment? Since the problems require students to think about something, the problems should reflect something worthwhile to wonder about and something

real. How does a classroom teacher create problems that fit these requirements around each issue and theme discussed in the curriculum. This set of questions surrounds designing problems that invite discussion.

Another set of questions concerns arranging a performance assessment within the traditional school structures. During a normal school day under normal conditions, I have to find a way to put together panels of judges I will need several times during the year. Where can I locate people? How can I begin to involve the community outside school? Also, without the benefit of the university personnel who work in our building,<sup>4</sup> how does a teacher put together these panels?

I feel very strongly about providing opportunities for performance assessments. The kind of information I received about each student and the reactions of the students make it clear to me that this is a much better method of assessing understanding than typical paper and pencil tests. If I can assess my students' understanding in a more realistic situation and at the same time increase their confidence in themselves in relationship to mathematics, how can I simply rely on only traditional tests?

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<sup>4</sup>The College of Education at Michigan State University has been in a partnership with our school since January 1989. This collaboration is aimed at enhancing the education of practicing professionals (at both institutions), prospective teachers, and high school students. Some of these individuals served as judges are part of our partnership work.

## References

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## **Appendix**

### Algebra 2 Discussion Final Guidelines

Please keep in mind that this is a new experience for the students as well as for us. Give the students plenty of opportunity to explain themselves but if it is obvious that they are trying to fake it or are unsure of themselves let them know that it is not what we are after and move on.

Only one student per problem. They have been instructed that they will have to discuss the problem on their own without help from other members of the group. After you feel this student is finished if you want to ask another student some questions about this problem that is fine.

If you pick a problem that a student seems unprepared for, let them do what they can and then come back to that student with a different problem. Please make note of this on the evaluation form.

Please use the following evaluation sheet in assessing the student's discussions. If you find the categories I have outlined unusable or too constraining, feel free to write comments in the comment section or on the back. In assigning the final points you need to be as specific in your comments as possible. Also remember that I will need these forms to discuss their evaluations for any student who wants to check on their performance. If possible, please inform the students of their score. If you can't due to time restrictions and opportunity to confer with the rest of the team, I will be available for them to check grades before school and after on Wednesday and Thursday.

Algebra 2  
Discussion Final

Name \_\_\_\_\_

Mathematics:

- |  |           |
|--|-----------|
| 1) Making sense of problem<br>(Understanding concepts) | 1 2 3 4 5 |
| 2) Problem-solving strategies<br>(Methods used)        | 1 2 3 4 5 |
| 3) Accuracy of results                                 | 1 2 3 4 5 |
| 4) Interpreting results<br>(What do the results mean?) | 1 2 3 4 5 |

Presentation:

- |  |           |
|--|-----------|
| 1) Ability to communicate results<br>(Clarity, use of charts/graphs) | 1 2 3 4 5 |
| 2) Explanation<br>(Able to answer questions)                         | 1 2 3 4 5 |

Overall Score

\_\_\_\_\_

Comments: