"THE BIG OLD CONVERSATION": REFLECTIONS ON MATHEMATICAL TASKS AND DISCOURSE

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> The teacher of mathematics should pose tasks that are based on
> - Sound and significant mathematics;
> - Knowledge of students' understandings, interests, and experiences;
> - Knowledge of the range of ways that diverse students learn mathematics; and that
> - Engage students' intellect;
> - Develop students' mathematical understandings and skills;
> - Stimulate students to make connections and develop a coherent framework for mathematical ideas;
> - Call for problem formulation, problem solving, and mathematical reasoning;
> - Promote communication about mathematics;
> - Represent mathematics as an ongoing human activity;
> - Display sensitivity to, and draw on, students' diverse background experiences and dispositions;
> - Promote the development of all students' dispositions to do mathematics.
>
> (NCTM 1991, p. 25)

> The teacher of mathematics should promote classroom discourse in which students
> - Listen to, respond to, and question the teacher and one another;
> - Use a variety of tools to reason, make connections, solve problems, and communicate;
> - Initiate problems and questions;
> - Make conjectures and present solutions;
> - Explore examples and counter examples to investigate a conjecture;
> - Try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
> - Rely on mathematical evidence and argument to determine validity.
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> (NCTM 1991, p. 45)

The vision of mathematics teaching that the National Council of Teachers of Mathematics (NCTM 1991) conjures up in *Professional Standards for Teaching Mathematics* is exciting. But what does it mean in practice? What sorts of problems and dilemmas await the classroom teacher who tries to create rich conversations about significant mathematical questions in an elementary classroom—conversations that engage and challenge *all* her students.

Inspired by the language of the *Standards*[1] and by the conversations we witnessed in the classrooms of a few colleagues,[2] we have been exploring with other teachers and teacher educators (see Featherstone, Pfeiffer, and Smith 1993) the new role of the teacher in classrooms in which students debate multiple solutions to problems, investigate problems for which they have no algorithmic solutions, and talk and listen as much to one another as to their teacher. During the winter of 1992, we also collaborated for five weeks in teaching mathematics in a third-grade classroom. This paper tells a bit about our work together and describes the development of our thinking about mathematical tasks and classroom discourse.

We are telling this story for two reasons. The first is connected to its actual substance: Our experiences in teaching this unit have led us to some conjectures about the relationship between mathematical tasks and discourse, and we put these forward in order to stimulate conversation with other educators. We believe that teachers struggling to teach math in new ways can help one another by sharing experiences and interpretations of that experience. Our second reason is of a somewhat different character: We see the collaboration underlying this story as a kind of "case" that is particularly relevant to the needs of the profession at this moment in the history of mathematics education, a moment in which the professional organization of mathematics teachers has persuaded a surprisingly large number of policy makers, teacher educators, and teachers that we need to teach mathematics in ways that few teachers have ever seen even on a videotape of "exemplary practice," much less experienced either as students or as teachers. We will say more about this reason in the next-to-last section of the paper.

Because our thinking arises directly from our reflections on work we have been doing in the classroom, we begin by introducing ourselves and the setting and circumstances in which we worked together.

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[1]*Standards* refers to *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and *Professional Standards for Teaching Mathematics* (NCTM 1991).

[2]Especially the third-grade classroom of Deborah Ball and the fifth-grade classroom of Magdalene Lampert. Both Ball and Lampert teach at Michigan State University; they also teach mathematics daily in a local public elementary school. For more on their thinking and teaching see Ball (1990) and Lampert (1986).

OUR BACKGROUND AND COLLABORATION

Several years ago, as Kathy, a primary-grade teacher in an urban school district, moved away from traditional approaches to teaching reading and writing and towards "whole language," she began to want to teach math differently also. After seeing a videotape of a mathematics class in which third-graders discussed their different solutions to a math problem, she began to try out new approaches and to seek out others who were changing the way they taught math. In 1991, she joined several math study groups, including one organized by Helen, and took a graduate course that investigated the implications of the NCTM *Standards*.

Helen teaches in the College of Education at Michigan State University and writes about teaching and learning. In 1991, as a part of her research on the teaching of mathematics, she organized Investigating Mathematics Teaching (IMT), a group of teachers who met regularly to discuss issues related to nontraditional mathematics teaching. The more she listened to the conversations in the IMT group and watched videotapes of children and teachers discussing mathematics, the more she wanted to work, and perhaps teach, in a classroom.

In January 1992, Kathy presented Helen with the opportunity she had been looking for. During the previous fall, the IMT group had focused its discussions around videotapes and other materials documenting teaching and learning in a third-grade classroom over the course of a unit on integers.[3] Although unpleasant memories of her own school experiences with negative numbers had prevented Kathy from enjoying discussions of this subject matter (and led her to half-resolve to keep well away from it in her own classroom), she was also attracted to the challenge of teaching her own students about negative numbers and to the thought that in doing so she would deepen and extend her own understandings of mathematics.

So when, during a math lesson, one of her third-graders asserted that "you can't take 9 from 0," and another quietly disagreed, Kathy decided to look for a way to take up the

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[3]These materials, which include videotapes, student and teacher journals, the field notes of graduate students observing the class, and interviews with students across the school year, were collected in 1989 and 1990 by Deborah Ball and Magdalene Lampert as part of the Mathematics and Teaching Through Hypermedia Project (Ball, Lampert, and Rosenberg 1991).

pedagogical gauntlet. She told Helen that she would like to teach her students about negative numbers but knew that she would need some assistance in order to do so. Helen offered to help her plan and teach such a unit.

In an effort to make an inherently abstract topic more concrete, we decided to use a thermometer to introduce below-zero numbers. (Mother Nature helped us to relate our discussions to the eight-year-olds' lives: A few days before our first lesson, the principal announced over the public-address system that he was cancelling recess because "the radio has just announced that, allowing for the wind-chill factor, the temperature this morning is minus 15 degrees.") Our ambitions were limited; we intended to show the students that this numeric territory exists and that, if you subtract a large number from a small one, you will end up there. We did not intend to pose any problems requiring subtractions or multiplication of negative integers; we did not expect questions about, for example, the meaning of 2 - (-6) to arise in discussions based around the thermometer. In fact, however, we revised our pedagogical itinerary several times.

MANAGING THE DISCOURSE

In late January, when we began to work together, Kathy's third-graders were already very much excited by math. Sometimes, math discussions would extend for over an hour as children sought and explored patterns, proposed experiments, and presented and examined theories. Their seriousness, their excitement, and their ideas delighted both of us.

However, all was not entirely well. After Helen entered the classroom, Kathy became concerned about the conversation: She felt that children were not always listening respectfully to one another. She talked with the third-graders about the problems; they explained that they often found it hard to wait for a chance to present their thinking. We continued to feel troubled, even though we believed that the difficulties arose precisely *because* the students were thinking hard and had interesting ideas that often carried the discussion in unexpected directions.

As we analyzed the events of the classroom together, we began to see connections between the mathematical tasks we posed for the group and the character of the ensuing discourse. We came to think about the mathematical tasks that we gave our students as falling into two categories: *convergent* and *divergent*.

When we talk about a *convergent* task, we mean one that tends to head students toward a single answer.[4] Along the way, there are always possibilities for many different ways of thinking and the development of different mathematical skills and concepts, but in the end, the group will settle on *an* answer. We used a convergent problem to introduce the children to the idea that there were numbers below zero: “The temperature in Anchorage, Alaska, was 2 degrees yesterday morning, but it fell 6 degrees by sunset. What was the temperature in Anchorage then? Can you write a number sentence to show what happened?” Children approached this task in a variety of ways. Luke, for example, wrote, “In the morning the temperature was 2° and then it dropped 6° so it was 4° below zero at night.” Mary, meanwhile, wrote “2 - 6 = -4” and Jonathon came to the board to explain that “if the higher number is first . . . all you would have to do is count how many more this [top] number is than this [bottom] number,” but that this pattern did not seem to hold for problems with answers *below* zero. But after some discussion, everyone agreed that the temperature in Anchorage must have been -4 degrees at evening and that “2 - 6 = -4” was one plausible representation of the change—though not the only one, since Luke’s sentence said the same thing. Thus, although the children approached this question in multiple ways, they *converged* on a single answer.

A *divergent* task has many answers—often, an infinite number. It can, therefore, lead children to explore many different avenues; sometimes, ones that head in quite unexpected directions. On our first day of work on temperature, we gave the third-graders a divergent problem that grew directly out of a class discussion of the day’s temperature. After introducing the thermometer and soliciting students’ ideas about it, we noted that the local temperature had been 30 degrees Fahrenheit at daybreak and that it had risen 5 degrees since then. The third-graders agreed that we could represent this change by writing 30 + 5 = 35. Children had then proposed other ways in which the temperature might have arrived at 35 degrees: It might, for example, have started at 39 degrees and dropped by 4 degrees. At the conclusion of this discussion, we told them to take out their math journals and “write number sentences that show some different ways to get to 35 degrees.” This task was *divergent* in the

[4]Some good convergent tasks begin with a divergent question. For example, “Paul had nickels, pennies, and dimes in his pocket. He pulls out two coins. How much money might he have?” poses a divergent problem. However, if we add, “How many different amounts are possible?” or “How will we know that we have them all?” we make the task more convergent.

sense that it allowed for an infinite number of answers. It might potentially have moved the class in many different directions.

In what follows, we will describe discussions that grew out of one convergent and one divergent task and reflect on what we saw happening in these and other mathematics lessons. Our first example comes from our third class.

A CONVERGENT TASK

On Tuesday, January 28, as the class generated ways to get to 35 degrees on the thermometer (32 + 3 = 35, 41 - 6 = 35, etc.), one student asserted that 60 - 25 = 35. Another disagreed, arguing that 60 - 25 = 45 because: "You can't take 5 from 0 so you write down the 5. Then 2 from 6 is 4."

$$\begin{array}{r} 60 \\ \underline{-\ 25} \\ 45 \end{array}$$

After discussion in which several of the third-graders suggested alternate ways to approach this problem and we pondered, not the first time, the difficulty of helping third-graders to extend what they have learned about regrouping from one context to another, Kathy asked the students to write in their journals about which number sentence they agreed with and why. On Thursday morning, she launched the math discussion by returning to what they had written. Here is her journal description of the ensuing conversation:

> Mary Liz came up to the board first and explained in great detail on the thermometer and with trading that she thought 35 was the answer. Janine [whose disjointed presentations often elicit cries of exasperation from her classmates] commented at length on her own presentation and was pretty coherent, counting in the thermometer and writing the problem and erasing and writing several times. She has a real need to be at the blackboard writing and erasing and drawing. Violet was next . . . She surprised me by illustrating her thinking with tally marks. She drew enough to illustrate 60 and then proceeded to erase the amount that represented 25. Everyone was taken with this strategy. Later someone else used this to explain another point . . .
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> Cindy [who had been struggling with regrouping since second grade] was very clear and explained how she got *35!* She was extremely confident, at one point saying "and 6 take away 2," and then correcting herself and saying, "No, I traded, so it's 5 take away 2 is 3." She also used the thermometer . . .

Then Nathaniel was on. He used the minicomputer[5] to explain his reasoning. But once wasn't enough; he showed two different ways to trade down on the minicomputer, both ending up with 35. He turned with a huge grin on his face and asked if there were any questions. There was some conversation here that led to him redoing the problem on the minicomputer. The discussion was about whether what he did was the same as what Mary Liz had done. I then said, "I'll write the numbers to go with what Nathaniel does on the minicomputer." I encouraged others to write with me. This seemed to be very helpful to the children. We did this slowly and talked it through. Most were writing. After that I sat down and Nate said (it seemed out of the blue) . . . "Remember when Cindy and I did the same thing?" and he looked around for the chart paper (which by some miracle was visible) and said, "Yeah, here, we did the same thing: 60 take away 49. We forgot to trade. You don't know which problems are going to be like that, you don't know which ones to trade. (He just kept talking like that, and we were all listening) How do you know? It's pretty hard . . . If you have *a big old conversation*, you can tell. I think I know. Do you know, Mrs. Beasley?"

I didn't want to say I did know or didn't know. I said something like, "I'm really interested in knowing how you think about it, Nathaniel." He then proceeded to explain that if you have a "dee" in the number you have to trade. [Everyone seemed to understand without explanation that he meant numbers like "twendee" (20), "thirdee" (30), and "sixdee" (60).] Children muttered questions. He kept going. He was having the time of his life. His confidence was growing right before our very eyes. He is pretty confident, but it was the confidence in his ideas that was taking grip of him. He wrote down "70 - 29" and proved on the minicomputer that you have to trade. Then he showed how you didn't have to trade with 48 - 22.

Violet asked him then, "Would you do the same thing with 95 - 60?" Nate said yes and did this:

$$\begin{array}{r} {}^{8}\not{9}5 \\ \underline{-60} \\ 25 \end{array}$$

Mary Liz spoke next and pointed out that you don't have to do trading. Nathaniel then said, "The dee has to be on top. That's what I think she's trying to say." Mary Liz explained the whole thing again. Nathaniel finished "his lesson" with a detailed demonstration/explanation of 80 - 27 and made the following conjecture: "You have to trade if you have a dee on the top. 70, 80, 50 . . ."

Two convergent questions—What is 60 - 25? How do you know whether you have to trade?—lie at the heart of this lesson. *We* posed the first one, lifting it from the children's

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[5] A teacher-made board which enables children to represent, with checkers, integers and operations with integers.

discussion. *Nathaniel* formulated the second, just at the moment at which the group appeared to have converged on an answer to the first. This lesson suggests how rich a convergent question can be; it also suggests some of the ways that its convergence can help the teacher in the complex task of orchestrating a class discussion.

As the manager and facilitator of these discussions, Kathy had much to do. She supported and encouraged children as they listened, thought about the question, and presented their ideas. She had to decide whether the discussion was moving in a useful direction, and if it was not, she had to decide how to intervene in a way that supported discourse and the development of children's mathematical ideas. She had to think about the needs of individual children, asking herself as she scanned the room who required some extra support in order to stay engaged, who needed a look or a word at the right moment.

However, the mathematical task made her job somewhat easier than it sometimes was. To begin with, it was a "good" question: It was challenging to the students, and it carried them into important mathematical territory. Furthermore, the convergence of the question directed the flow of the discussion. Since each child was trying to figure out, first, "What is 60 - 25?" and, then, "When do you trade?" all the ideas presented were related, either reinforcing or challenging each child's thinking in a way that allowed individuals to continue along their own pathways. Although many children wanted a turn at the board, they were focused and thoughtful as different classmates explained their ideas. The fact that children often saw "their" idea being presented relieved some of the pressures on turn-taking.

Although the children were not yet completely clear about when regrouping is required, we were both pleased with the quality and direction of this discussion. Reflecting the next day about the conversation we had about this lesson, Helen wrote in her journal, "Both of us felt optimistic that the kids would work out the conditions under which regrouping was necessary in subtraction with a little more time." Both of us felt that the discussion itself was valuable. We both agreed that:

> The discussions we are having about when you regroup, or how you know whether you have to regroup, are worthwhile in themselves over and above their contributions to kids' understanding; They are worthwhile because they are challenging to just about everyone, and because each person's contribution adds something to the thinking of the group. They are also worthwhile because they are enhancing students' understanding of important ideas in mathematics.

The students remained the leaders in their learning. They were headed toward the right answer, but they were discovering the way themselves. The focus remained clear, but there was considerable room for a variety of theories and concepts. Subtraction skills were integral to the discussion as the children struggled with this complex mathematical idea.

A DIVERGENT PROBLEM

After the children discovered and agreed that "we need to trade when the lower right-hand number is higher than the upper right-hand number," we spent several days exploring the interrelationships between the different ways that this group of third-graders had discovered for representing two-digit subtraction. On February 20, we began to look at numbers below zero. Our February 20 and February 21 classes offer glimpses of the third-graders at work on a *divergent* problem and of the somewhat different challenges and opportunities that this sort of mathematical task offered them.

After the children had worked together on the problem involving temperature changes in Anchorage and agreed that 2 - 6 = -4, Lisa proposed a conjecture: "In subtraction, if the second number is bigger than the first number, the answer will be below zero." Her suggestion seemed to intrigue many of the third-graders, but there were several who did not, at first, understand what she was saying. Although we might very well have asked the children to investigate Lisa's conjecture during their work time, we felt that a more general exploration of this new numeric territory would give more children access to the idea of negative numbers, while allowing those who were interested to examine the merits of Lisa's conjecture. So Kathy directed students to "Write as many number sentences as possible that end below zero."

A videotape of this lesson shows a class of exuberantly engaged eight-year-olds working hard, both alone and in small groups. Several move back and forth between their desks and the large thermometer that Kathy had drawn on poster paper and taped to the chalkboard. Two children are counting aloud, writing number sentences together in their notebooks. Mary Liz is making a chart on the board, showing her many calculations involving one- and two-digit numbers. Nearby, Violet writes "1000 - 2000 = -1000." Cindy, who regularly antagonizes her classmates while simultaneously struggling for acceptance, looks unusually relaxed and cheerful; after conferring in whispers with Tina, she takes a place next to Violet on the board and begins to write:

80	5
-83	-6
-3	-1

Even though the room is noisy, the children are clearly pursuing mathematical ideas; the two teachers look radiant. Helen wrote in her journal that night:

> This writing period was electric with activity: Lots of ideas were popping up, kids were going back and forth to the big thermometer to check their calculations . . . I was working with Martha [whose limited skills often seem to prevent her from joining her classmates mathematical conversations] who soon set herself a manageable and appropriate task: She started writing number sentences equal to 1. I was delighted: As Kathy says, Martha is often off in "answerland." It seemed wonderful that there was a space for her to do a task that made sense to her.

When Kathy called the class together after half an hour of this independent work, the children volunteered eagerly to share the results of their thoughts and calculations. Because the question had provided them with many points of entry, each child had found a way to engage in mathematical thinking. One started with 23 - 9 = 14 and then moved to 9 - 23 = -14. Several made charts with 20 to 30 examples of this idea. A number of children made conjectures after generating some examples. Here are four that we were able to discuss as a group in the days that followed:

> Cindy:
> If you have a problem that is like Lucy's (11 - 9 = 2) and the answer is above zero, if you switch it around (9 - 11) you'll have an answer below zero.
> 7 - 4 = 3   3 - 7 = -4
>
> Jonathon and Luke:
> When you have a problem like 9 + 4 = 13, then just change the plus sign to a minus sign and the second number to the answer, only below zero, and the answer to the second number.[6]
>
> Violet:

[6]In other words, 9 + 4 = 13 implies that 9 - 13 = -4. This conjecture is awkward, but the children who created it managed to make its meaning clear to the rest of the class *and to the teachers*.

A great big number with a lot of zeroes can be easier than a smaller number without zeroes.

George, Noah, and Justin:
If you are subtracting and you have a zero in the one's place in the top number and the same number in the ten's place for both numbers, the answer will be the negative of the number in the one's place on the bottom.

80	60
<u>-85</u>	<u>-62</u>
-5	-2

The children recorded many more examples of mathematical thinking in their notebooks and looked forward to explaining them to the rest of the class. When we concluded the math discussion twenty minutes later, we promised that the next day we would give turns to those who had not yet gotten a chance to share.

The next day, all the children had much to say about their thinking. But, although most were willing to listen to each other and to think about the mathematical ideas before them, several had trouble concentrating. Attention often strayed from the student at the board: Some children were talking to each other about their ideas or even about something unrelated to math. Some asked impatiently when they would get *their* turn. Children were drawing on a diverse assortment of math skills and concepts and displaying some spectacular mathematical thinking, but everyone had a different agenda. The group focus was missing.

Kathy's role was also changed. She was still in charge of orchestrating the discussion, but the focus was now much broader: Students had to try to understand whatever mathematical idea or question was being presented at the moment. The richness and diversity of thinking was impressive, but for a number of the eight-year-olds, it was overpowering. Because many seemed to have lost their way, it was hard for Kathy to listen with the concentration she needed in order to understand each student's ideas and keep an eye on all the other factors involved in orchestrating discussions.

Although the third-graders had done extraordinary work on this open-ended task during their independent work time, we felt dismayed by the ensuing discussion. The students did not seem to be very interested in one another's ideas, and we did not feel that they were learning as much as usual from this period of joint deliberation. Furthermore, many seemed irritated—frustrated by having to wait a long time for their own moment at the board and hurt that, when their turn finally came, their classmates attended less closely than usual to their ideas. It seemed to us, as we analyzed the lesson, that the very open-endedness that provided children with so many points of entry and stimulated so much interesting mathematical theorizing had undermined the group discussion. A few days after that lesson, Kathy reflected in her journal, “The discussion that emerges from a very open-ended question is much more out of my control . . . Maybe the assignment for the journal writing was too open-ended.”

We did not, however, stay with this conclusion for long. As we continued to talk and write about what was happening in the classroom, and to watch children develop their ideas and conjectures, we saw that students had gotten vast intellectual mileage out of our open-ended task. Indeed, we believe that the excitement generated by the relatively unstructured period of exploration of this divergent question served important educational purposes: It gave these eight-year-olds a chance to “play” in a new mathematical domain and to generate number theory; it taught them that, in relation to this relatively abstract domain of mathematics, they were powerful mathematicians; it helped them to grow comfortable with numbers below zero. Over the next few weeks, as the third-graders continued to work with integers, Kathy often exclaimed incredulously, “I can’t believe how easy this is for them.” Her own experiences with negative numbers had been far less positive. Indeed, even as an adult, she felt somewhat uncomfortable with the concept. While not all of the third-graders generated theories about these numbers, none of them seemed at all frightened in the new territory, and all seemed to enjoy the work they did there. One of Peter’s journal entries captures for us the playful spirit that enlivened this work: In the midst of a list of number sentences equalling zero, he wrote
“- Pat + Pat = 0,” making oblique reference to a recent visit to the school of author/illustrator Pat Cummings.

In some ways, then, we would say that as we struggled with the children’s impatience during the February 21 class, we were suffering the natural consequences of creating a highly divergent *and enormously exciting* task. It seemed to us that the impatience that the children felt grew out of their excitement: They were too excited about their own discoveries and the prospect of presenting them to classmates to listen quietly when their classmates described what they had seen on somewhat different mathematical paths. To put the matter paradoxically, the success of the task in generating excitement and powerful thinking laid the seeds for difficulty in the ensuing conversation.

We came to feel that, at least in the particular classroom in which we were working, there were no perfect tasks which would serve all our pedagogical aims. Like Magdalene Lampert (1985), we believe that the teacher must continually juggle competing goals, adopting means which further some of her purposes, while continually evaluating the price she may be paying in other areas. Rather than saying that convergent questions are good and divergent questions are bad, or vice versa, we wish to suggest that good convergent questions and good divergent questions may be useful for different things.

A good *divergent* question may be particularly helpful in showing a teacher where her students are. It may also encourage them to generate theories and to begin to see themselves as mathematical thinkers. We believe that, in the case we have just described, the third-graders in Kathy's class had powerful experiences as they experimented with calculations involving negative numbers and looked for patterns. They saw themselves as discoverers; they experienced mathematics as play and exploration. In addition, a divergent task can invite children to set their own challenges. For example, while her classmates subtracted large positive integers from smaller ones, Linette worked on a way to extend an observation about positive numbers into the new territory below zero: She knew that if 9 - 5 = 4, then 4 + 5 = 9; did that mean, she wondered that because
4 - 6 = -2, -2 + 6 = 4? A good divergent question may, as its label suggests, open up mathematics.

A good *convergent* question may create a situation in which students are particularly likely to build on one another's ideas; if so, it may help them to value group discussions more. Convergent questions may be particularly helpful to students and teachers who are struggling to establish norms and expectations for good mathematical discourse. We want to refer back to the discussions we have already described to clarify this conjecture. When Nathaniel explained his ideas about "dee" numbers, his classmates listened intrigued. They were interested in part because, like him, they were struggling to figure out how they could tell whether they needed to regroup in order to solve a subtraction problem. This was the sort of conversation that convinced them that math discussions were worth listening to, that norms to facilitate these discussions were worth establishing and maintaining. On the other hand, Linette's conjecture about *adding* a negative number to a positive one was far off the path that the other third-graders were traveling; when she presented it, many acted less engaged. Although Kathy could compel students' quiet and even attention, the discourse itself provided fewer intrinsic incentives to listen.

MATHEMATICAL DISCOURSE AND THE CLASSROOM ECOSYSTEM

We wrote and talked a great deal about our pedagogical options in late February, and as we continued to exchange ideas, we began to see that the norms of the classroom help to determine the consequences of giving students particular mathematical tasks. The divergent task posed dilemmas for us partly because students had come to expect a public forum for their stories, feelings, and ideas. Because Kathy believes that the children's confidence that she and their classmates will "hear" them when they need to be heard is an essential foundation of this learning community, she felt committed to giving a turn to each child who wanted to share a journal entry. In consequence, a well-chosen divergent task generated a long queue of children who wanted and expected to share sometimes unrelated ideas.

In a different classroom with different norms, the discussion period might have looked very different. For example, one September afternoon, we observed another class of third-graders as they discussed the number sentences they had generated that equalled 10. Although the assignment had been highly divergent, the students focused uncomplainingly on an extended discussion growing out of one number sentence; during this thirty-minute conversation, not one child asked restlessly for a chance to write *her* sentences on the board. It was early in the school year, and the eight-year-olds may still have been trying to figure out their new teacher. Whatever the reason, the different norms and expectations in this classroom clearly created different behaviors. Teachers and students shape classroom norms together in ways that respond to different needs, both psychological and institutional, and serve a wide variety of different purposes. And it is within the framework of these norms and these multiple, often conflicting, necessities that teachers must think about the pros and cons of particular mathematical tasks.

In Kathy's third grade, part of the payoff for independent work was the joy and excitement of presenting one's thoughts to the group. There seemed to be two ways that children expected their presentation to proceed. First, they wrote and drew examples that clarified their ideas and, second, they were asked questions that required them to listen carefully to another child and to figure out a way to respond that satisfied their questioner. They valued both parts of the presentation highly. Over and over again, we saw children listening thoughtfully to another eight-year-old who was challenging their approach to a problem, asking questions of their challenger in ways that indicated that they were trying to understand the challenger's thinking rather than to refute his point. We saw them working hard to break out of the prison of their own perspective on a problem in order to make sense of someone else's ideas. Nobody in the classroom would willingly give up the opportunity to engage in this kind of thinking and dialogue and so, when the mathematical task took children in many different directions, Kathy was faced with a monumental task of orchestrating discussions that were so diverse, complex, and numerous that they became overwhelming. For us, the dilemma was how to satisfy the norms of discussion within the very real constraints of a limited school year and third-graders' limited capacity to concentrate and refocus.

LEARNING FROM WRITING TOGETHER

By the end of the school year, we had arrived at this assessment of the costs and benefits of different sorts of mathematical tasks. Wondering how others were thinking about the relationship between tasks and discourse, we decided to try to write about our ideas and the experiences that had created them. We revisited our journals and videotapes. We wrote and rewrote. Looking at ourselves and the students from a bit of a distance allowed us to see the dilemmas of task and discourse in new ways. After struggling with the organizational problems in our third draft, Kathy wrote Helen a note:

> As I was writing about the dilemma of orchestrating a divergent discussion, I got to thinking that just as this is a "new way to teach math" I need a new structure to teach math. Think about the video of that Friday lesson that I aborted [in order to talk with the children about the frustration that seemed to hang in the air] and the class and I talked about ways to solve the problem. The children offered a new structure. Traditionally it is the teacher talking to the whole class. We had moved to it being the teacher or a child talking to the whole group. Now the children were saying, "Create a different structure for time like this." For times when they have explored a divergent question they wanted a divergent structure: Children would choose the idea they were most interested in and explore it with a small group of equally interested children.
>
> I could see this working. I could collect notebooks to see how many different ideas there were, group similar ideas, and have these children put them on chart paper to give a limited explanation, with the understanding that they would choose the idea they were most interested in and explore it to their

> satisfaction in the smaller group. There would be some problems of peer pressure at work here, but still I think that, with some time to work out the kinks, this would work. I am really excited about this idea: Divergent discourse simply calls for a nontraditional structure. It also requires the teacher to play a new role, orchestrating groups of children discussing a variety of mathematical ideas rather than a whole group of children discussing one idea.

Our discussion of this idea led us back to the goals we had articulated when we started our work together. We knew that our students did not "have" to know about negative numbers in order to negotiate fourth grade; we were introducing them to negative integers in order to allow them to *engage* with theoretical mathematics, to generate theories in a new realm, to feel their own power as mathematicians, to see a new numeric territory, and to explore the operation of subtraction more freely. The divergent structure that the students had suggested as an appropriate follow-up to their independent work on a divergent problem seemed very appropriate to these goals.

REFLECTIONS ON LEARNING: THEIRS AND OURS

We have told this story for two reasons: We want to contribute to a larger conversation among teachers, math educators, and others about the kinds of mathematics teaching that the NCTM advocates in the *Standards*; we also want to make an argument for the kind of collaborative teaching and inquiry that we ourselves engaged in for over five weeks in 1992.

The vision of the NCTM represents a major break from conventional practice. Over the course of the last twenty years, many elementary school teachers have made "manipulatives"—from bottle caps to Dienes blocks—a part of their mathematics program. Teachers who want to move in that direction can draw on a rich stock of teacher lore; colleagues and magazines can advise on everything from how to store small objects to when and how to introduce place-value boards. But teachers who read the *Standards* and become excited about the possibilities for teaching through conversation about meaty problems have many fewer resources. The NCTM vision is not warmed-over progressive education; it is not the New Math of the late 1950s and 1960s; the classrooms NCTM describes are unlike any most teachers have ever seen. In consequence, there is little teacher lore to assist the teacher who struggles to create worthwhile mathematical tasks, to establish norms that will support mathematical discourse that is educative simultaneously to the most and least confident children, and to assess learning in authentic ways.

Nor are math educators in a strong position to provide the lore that will support pioneering teachers; nationwide, only a handful have done this kind of teaching themselves. Like most of our colleagues in schools and universities, we are relative newcomers to this kind of teaching. But we believe that in working together we learned more about the dilemmas and the opportunities that it offers children and their teachers than we could have learned alone even over a considerably longer time. We believe that this case of our learning demonstrates the value of joint work.

We see a close connection between our own learning and that of the students in Kathy’s third grade. Like them, we believe that we learned from conversation and from writing. We learned from looking together at the same phenomena, by talking and writing about something we both had seen, and from struggling to make sense of another person’s perspective on an event we had both witnessed. We learned because our collaboration forced us to write and talk—you cannot teach together without talking. Because there were two of us in the classroom, we each saw more than we could have if we had been in the classroom alone (we probably also taught better than either of us could have taught alone).

We believe that the sort of partnership we established, one in which both parties talk daily about their teaching and exchange detailed teaching journals, offer possibilities not only for the learning of the participants but also for the creation of more “teacher lore” about this kind of mathematics teaching. We are enthusiasts; we hope we can sell our enthusiasm to others.

EPILOGUE

Interestingly, we concluded our unit on below-zero numbers in a way that seemed very much in the spirit of the structure that the students had suggested several weeks earlier. When Kathy announced that we would move on, many seemed outraged: “What about our conjectures?” they asked indignantly. But when Kathy suggested that she would leave the conjectures up on the bulletin board so that they could continue to work on them at other times of the day, all seemed satisfied. Children continued to explore conjectures about operations involving negative integers for several weeks, even after they had, as a group, moved into new mathematical territory. Their interest in continuing to investigate these ideas even when they knew that they would not be presenting the results of their research to the full class suggested to us that many of these eight-year-olds found these opportunities to theorize deeply engaging. Their ongoing interest in these ideas strengthened our conviction that the free exploration that had generated these conjectures had offered them a chance to experience their own power as mathematicians. And it strengthened our confidence in the value of what Nathaniel had called “a big old conversation.”

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