$4.00

Elementary Subjects Center
Series No. 48

WHAT DOES CGI MEAN TO YOU? TEACHERS' IDEAS OF A RESEARCH-BASED INTERVENTION FOUR YEARS LATER

Nancy F. Knapp and Penelope L. Peterson

Published by

The Center for the Learning and Teaching of Elementary Subjects
Institute for Research on Teaching
252 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

August 1991

This work is sponsored in part by the Center for the Learning and Teaching of Elementary Subjects, Institute for Research on Teaching, Michigan State University. The Center for the Learning and Teaching of Elementary Subjects is funded primarily by the Office of Educational Research and Improvement, U.S. Department of Education. The opinions expressed in this publication do not necessarily reflect the position, policy, or endorsement of the Office or Department (Cooperative Agreement No. G0087C0226).

Center for the Learning and Teaching of Elementary Subjects

The Center for the Learning and Teaching of Elementary Subjects was awarded to Michigan State University in 1987 after a nationwide competition. Funded by the Office of Educational Research and Improvement, U.S. Department of Education, the Elementary Subjects Center is a major project housed in the Institute for Research on Teaching (IRT). The program focuses on conceptual understanding, higher order thinking, and problem solving in elementary school teaching of mathematics, science, social studies, literature, and the arts. Center researchers are identifying exemplary curriculum, instruction, and evaluation practices in the teaching of these school subjects; studying these practices to build new hypotheses about how the effectiveness of elementary schools can be improved; testing these hypotheses through school-based research; and making specific recommendations for the improvement of school policies, instructional materials, assessment procedures, and teaching practices. Research questions include, What content should be taught when teaching these subjects for understanding and use of knowledge? How do teachers concentrate their teaching to use their limited resources best? and In what ways is good teaching subject matter- specific?

The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

The findings of Center research are published by the IRT in the Elementary Subjects Center Series. Information about the Center is included in the IRT Communication Quarterly (a newsletter for practitioners) and in lists and catalogs of IRT publications. For more information, to receive a list or catalog, or to be placed on the IRT mailing list to receive the newsletter, please write to the Editor, Institute for Research on Teaching, 252 Erickson Hall, Michigan State University, East Lansing, Michigan 48824-1034.

Abstract

Interviews were conducted with 20 primary teachers who, three or four years earlier, had participated in inservice workshops on Cognitively Guided Instruction (CGI)--a research-based approach that emphasizes using children's mathematical knowledge to teach mathematics. Although all but one teacher were still using CGI to teach mathematics, their use varied widely from occasionally or supplementally to mainly or solely. Teachers' use was significantly related to ideas about what it means to "know" mathematics, how students learn mathematics, and what responsibilities and roles teachers and students have in learning mathematics. Three patterns of change in CGI use emerged: Group 1 teachers reported a steady, gradual increase to reach their current main or sole use of CGI; Group 2 reported having never used CGI more than supplementally or occasionally and were settled in that use; Group 3 reported using CGI in earlier years but now using it only supplementally or occasionally. These patterns of use were related to the meanings that teachers had constructed for CGI. In their espoused beliefs and practices, Group 1 described CGI conceptually; Group 2 described CGI procedurally, as using manipulatives or word problems; while Group 3 showed a marked incongruity between their espoused beliefs and espoused practices.

WHAT DOES CGI MEAN TO YOU?
TEACHERS' IDEAS OF A RESEARCH-BASED INTERVENTION FOUR YEARS LATER[1]

Nancy F. Knapp and Penelope L. Peterson[2]

A major issue confronting the educational community today is whether and how researchers and teacher educators might assist experienced practicing teachers in reforming their classroom practice. While the leaders of the mathematics education reform movement have articulated new visions of mathematics and mathematics teaching and learning (Mathematical Sciences Education Board [MSEB], 1989, 1990; National Council of Teachers of Mathematics [NCTM], 1989, 1991; Steen, 1990), these visions continue to be far removed from the reality of teachers' work and quite different from their current practice (Ball, 1990a; Cohen, 1990; Peterson, 1990; Peterson, Putnam, Vredevoogd, & Reineke, in press; Wiemers, 1990; Wilson, 1990). Part of the disparity between the visions and reality lies in the disparity between the knowledge and beliefs of the community of researchers and reformers and the knowledge and beliefs of practicing teachers.

For the last five years, the community of mathematics education researchers and reformers has been working together systematically on the development of a set of shared understandings and beliefs about mathematics, mathematics learning, and mathematics teaching that contrasts sharply with those traditionally held. These developing understandings and beliefs involve views of mathematics as a science of patterns in which reasoning and "proof" are the tests of truth and as a body of developing knowledge that is growing and changing rather than

---

[1]This paper was originally presented at the annual meeting of the American Educational Research Association, Chicago, April 1991.

[2]Nancy F. Knapp, a doctoral candidate in teacher education at Michigan State University, is a research assistant with the Center for the Learning and Teaching of Elementary Subjects. Penelope L. Peterson, professor of educational psychology and teacher education at MSU, is the co-director of the Center. The authors thank the twenty teachers whose views and thoughts are described in this paper for their willingness to take the time out of their busy lives to answer questions. The authors appreciate their thoughtfulness and their candor about their failures as well as their successes and their insights about the dilemmas and challenges that they face daily in their teaching. The authors also thank Jere Brophy and Glenda Lappan for their comments on an earlier draft of this paper. The order of authorship on this paper is alphabetical; the authors view this paper as a collaborative effort to which they both contributed in important ways.

remaining static and fixed (MSEB, 1990); views of mathematics learning as involving the construction of mathematical knowledge rather than the transmission of mathematical knowledge (MSEB, 1989, 1990); and views of teaching as involving students in mathematical reasoning and in talking about and solving complex problems, as individual mathematical thinkers working as part of a community of mathematical thinkers (NCTM, 1991). For researchers, this shared vision is interconnected with theory and knowledge developed from research conducted over the last decade which has moved away from a behavioral paradigm toward alternative perspectives on what it means to know and understand mathematics (see, for example, Confrey, 1990; Putnam, Lampert, & Peterson, 1990; Resnick, 1989; Romberg, 1990; Sowder, 1989).

While a few members of the community of mathematics teachers have been involved in developing these visions, for the most part, teachers have not been involved in the discourse among researchers and reformers about these new views nor have researchers attempted systematically to share their reasoning and research evidence for these new views with teachers. Thus, while reformers and researchers have been involved in developing new knowledge and beliefs about mathematics learning and teaching, they have done so without teachers. Yet a key to the success of this reform involves knowledge growth in teaching (Shulman, 1987), and the hoped-for reforms will be mediated through the hearts and minds of teachers (Cohen, 1989; Elmore & McLaughlin, 1988). Indeed, most previous reform attempts in mathematics education are now judged to have failed primarily because researchers and curriculum developers failed to take into account the existing knowledge, beliefs, values, and purposes of teachers (see, for example, Clark & Peterson, 1986; Romberg & Carpenter, 1986) and of the cultures and contexts in which teachers work. For example, Stephens (1982) documented such a situation in his study of teachers implementing and teaching *Developing Mathematical Processes (DMP)* (Romberg, Harvey, Moser, & Montgomery, 1974), an innovative, activity-based elementary mathematics program developed by researchers at the Wisconsin Research and Development Center in the 1970s.

Stephens (1982) sought to determine what meaning had been given to knowing and doing mathematics in several classrooms where elementary teachers were using *DMP* to teach mathematics. He described *DMP* as a program using measurement as a tool for mathematical modeling, based on a constructivist approach to mathematical learning that assumed that children should begin with concrete experiences and move to the abstract. As Stephens saw it,

> *DMP* was intended to reshape conceptions of mathematical knowledge and school work. It sought to create a pedagogy in which children would be active in the creating and testing of mathematical knowledge. It saw mathematical inquiry as requiring exploration, investigation, choice and judgment. It believed that children could be assisted by their teacher to approach mathematical inquiry in this spirit. (pp. 243-244)

Stephens argued that *DMP* failed to achieve these purposes because "the implementation of *DMP* was assimilated into an existing network of beliefs, purposes, and values derived from a management perspective of instruction . . . where the focus of instruction was on the efficient transmission of a fixed body of subject matter to the children who comprised the class group" (pp. 220-221). This management perspective was embedded in the knowledge, beliefs, and thinking of the teachers as well as in the context of instruction and the culture of the school. Stephens also criticized the developers for taking a "center-out" approach to curriculum development which separated the work of teachers from the creation and testing of mathematics curriculum.

In 1985 three researchers at the Wisconsin Research and Development Center--Thomas Carpenter, Elizabeth Fennema, and Penelope Peterson--began yet another "center-out" approach to changing first-grade teachers' mathematics practice in ways that might follow from neoconstructivist learning theory and research conducted in the late 1970s and early 1980s on the development of children's problem solving knowledge in addition and subtraction (Carpenter, Moser, & Romberg, 1982). Building upon the knowledge and experiences of Center researchers in developing *DMP* and in working with teachers to use *DMP*, these researchers decided to try a different tack.

The developers of *DMP* attempted to change teachers' knowledge, beliefs, thinking, and practice by giving the teachers a mathematics curriculum program that had embedded within it the

researchers' own knowledge, thinking, and beliefs about mathematics and about mathematics learning. However, they did not make these views or the evidence for them explicit to the teachers, and they attempted to construct for the teachers the pedagogy that would facilitate students' development of the kinds of mathematical thinking they deemed desirable.

In their approach, called Cognitively Guided Instruction (CGI), Carpenter, Fennema, and Peterson made explicit the actual evidence for their own constructivist views of mathematics learning by sharing with teachers the research-based knowledge that Carpenter had gained about children's extensive knowledge and abilities to solve addition and subtraction problems before they even enter school. Rather than presenting this research-based knowledge to teachers as decontextualized principles or conclusions, the researchers presented teachers with the actual data from Carpenter's study by showing them videotapes of five-year-old children solving various types of addition and subtraction word problems, including those types that most adults have believed young children incapable of solving. They also shared with teachers two frameworks or sets of ideas constructed by Carpenter (1985) from these data. One framework described the 11 addition/subtraction word problem types as children think about them; the other presented the several kinds of strategies that children tend to develop to solve these problems as they progress from using concrete modeling and counting strategies toward using their knowledge of remembered addition and subtraction number facts. Also, rather than presenting the teachers with either a curriculum program or a preconceived and prepackaged set of instructional procedures, the researchers attempted to work with the teachers to figure out how this new research-based knowledge of children's mathematics learning and problem solving in addition and subtraction might be useful in each teacher's classroom practice.

At the beginning of their National Science Foundation-supported project, Fennema, Carpenter, and Peterson recruited 40 first-grade teachers from the Madison, Wisconsin, area with whom to work. With the understanding and agreement of the teachers, the researchers assigned them randomly to either an experimental group who experienced the month-long CGI workshop the summer of 1986 or a control group who experienced the same workshop the summer of

1987. During the workshop (for which the teachers received university credit), the teachers were given access to the framework of problem types and the related children's solution strategies through a set of readings written by Fennema and Carpenter, presentations by Carpenter, and class discussion. Teachers viewed videotapes of children solving addition/subtraction word problems until the teachers could identify both the problem types and strategies with relative ease. Teachers also interviewed five- and six-year-old children to see for themselves whether children actually used the solution strategies that had been discussed. As the researchers describe it,

> We did not tell teachers what to do with the knowledge they had gained. We discussed the importance of a teacher's knowledge of how each child solves problems; the place of drill on number facts; and the necessity for children to think and talk about their own problem solutions to each other and to the teacher. We talked about adapting the problems (by type of problem or size of number in the problem) given to a child depending on what the child understands and can do. We discussed writing problems around themes related to children's lives and classroom activities. We gave the teachers time to plan how they would use their new knowledge in their classrooms during the following year. Teachers talked extensively with us and with other teachers about possible implications of the knowledge about addition and subtraction. Most teachers wrote examples of all of the problem types to use in their classrooms and tentatively planned one unit that they would teach sometime during the school year. (Peterson, Fennema, & Carpenter, in press, p. 10)

Since the initial workshop, the researchers have written extensively about their findings related to teachers' use of CGI and the influence of CGI on teachers' knowledge, beliefs, and practices and on their children's problem solving in addition and subtraction and knowledge of number facts (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Peterson, Fennema, Carpenter, & Loef, 1989). After the first year, they concluded that teachers who experienced the CGI workshop during the summer of 1986 and used CGI in their teaching the following year had changed their beliefs to be more in line with CGI ideas. More specifically, when compared with the control teachers, CGI teachers agreed more strongly on a written survey with the following ideas:

- children construct rather than receive mathematical knowledge;
- mathematics instruction should facilitate children's construction of knowledge rather than teachers' presentation of knowledge;
- mathematics instruction should build on children's knowledge and understanding, and the development of mathematical ideas in children should provide the basis for sequencing topics for instruction;

- number facts should be learned within the context of problem solving and as interrelated with understanding.

When compared with control teachers, CGI teachers spent significantly more time on word problem solving, and they spent significantly less time drilling on addition and subtraction number facts. CGI teachers encouraged their students to solve problems in many different ways, listened more to their students' verbalizations of ways they solved problems, and knew more about their individual student's mathematical thinking. At the end of the 1986-87 school year, students in CGI teachers' classes did better than students in control teachers' classes on written and interview measures of problem solving and number fact knowledge.

We conducted the present study because we wondered how these changes weathered the test of time. Whatever changes inservice programs may accomplish, the lessons learned in inservices are usually interwoven with lessons learned elsewhere--through other inservices, curricular and administrative mandates, and imperatives from communities and parents. We wanted to find out what teachers thought about CGI now--three or four years after participating in the month-long summer workshop. We were curious about several issues. Did teachers' self-described practice indicate that they were currently using CGI in their elementary mathematics teaching, and if so, how? Would noticeable variations exist in the ways teachers described their development and use of CGI? And if so, what might account for these differences? Finally, we wondered if analyzing the meaning that teachers make of CGI and the understandings that teachers have developed would suggest to us possible explanations for these differences, and if so, we wanted to pursue them.

The methodology we employed in this study was entirely teachers' self-report. We did not observe what the teachers did in their classrooms, although we did ask them to describe to us in detail what they did on a typical day in their mathematics teaching. Thus, we might be subject to what Bruner (1990) has noted as the charge that "what people say is not necessarily what they do." But, as Bruner goes on to point out, there is a "curious twist" to this charge in that "it implies that what people *do* is more important, more 'real,' than what they *say*, or that the latter is important

only for what it can reveal about the former. It is as if the psychologist wanted to wash his hands altogether of mental states and their organization, as if to assert that 'saying,' after all, is only about what one thinks, feels, believes, experiences" (p. 16). In the present study, we took Bruner's ideas seriously and followed his urging that psychology stop trying to "meaning free" in its system of explanation. We explored in depth what teachers meant by CGI.

Method

Participants

The participants in this study were 20 teachers who had participated in month-long workshops on CGI as part of the large-scale study described above. Ten of the teachers had participated in the experimental group, and 10 of the teachers had participated in the control group in the original study. The experimental group completed the workshop in July 1986, and the control group completed the workshop in July 1987. All teachers in the present study were white females who taught in 18 different elementary schools either in Madison, Wisconsin, or in small towns within a 30-mile radius of the city. Sixteen of these schools were public schools, and two were private, Catholic schools. The percentage of minority students in each of these schools ranged from 0% to 28%, with a mean of 10.4%. The percentage of economically disadvantaged students in each of the public schools, as measured by those eligible for free or reduced lunches, ranged from 3.6% to 43.4%, with a mean of 19.8%. At the time of the initial study, all teachers taught either first grade or first/second grade. At the time of the present study, 15 teachers taught first grade, 3 teachers taught first/second grade, 1 teacher taught second grade, and 1 teacher taught second/third grade. The number of students in teachers' classes ranged from 16 to 27, with an average of 22 students per class.

To identify teachers for this study, we contacted the 40 teachers from the original study by mail in April 1990, explained the study, and asked them to participate in a one-hour telephone interview. We also made follow-up phone calls to those teachers who did not reply to our letter. We were unable to contact four of these teachers because they had left the district. Two teachers were no longer teaching mathematics at the primary level. One teacher consented to be

interviewed but was later unreachable. Nine teachers decided not to participate; of those we were able to contact for a reason, three said they did not have time to be interviewed, and two indicated they had never used CGI much and did not feel qualified to talk about it. Only one teacher who did not use CGI at all consented to be interviewed, and we suspect that there were others in this category among the teachers who did not consent. We feel our study would be more representative if we had been able to persuade those teachers to participate, but were limited by the consensual nature of all research studies that involve humans as subjects or participants.

Interview Questions and Procedures

Each teacher was interviewed by phone for approximately one hour by either Knapp or Peterson. With the exception of one interview that was conducted later, these interviews occurred between May and October of 1990. The interview protocol, attached as an appendix, specified 23 major questions to be asked and indicated possible probes. However, the interviewer did not ask these questions in a rigidly prestructured format. Instead, she followed the teacher's lead in the order in which questions were addressed. She also deviated from the protocol when she felt that she did not understand something that the teacher had said or when she wanted to determine the underlying meaning of a word, term, or idea that the teacher used during the interview. In such circumstances the interviewer sought clarification by asking the teacher what she meant or by asking the teacher to give an example.

Analyses

All interviews were recorded and transcribed verbatim for later analysis. We listened to each audiotape and edited and verified the transcripts so that all the dialogue that appeared in the transcript corresponded to the interview dialogue on the audiotape. We then conducted qualitative content analyses of the interview transcripts, using as our main tool *Hyperqual*--a Hypercard-based program developed by Dr. Raymond Padilla of Arizona State University. This program facilitates sorting, grouping, regrouping, and analysis of qualitative data such as the text data from our interviews. We developed the following 13 analytic questions which were used to

frame and guide our analyses; question numbers in parentheses (Q) refer to original protocol question(s) in the appendix which elicited the main information for each analytic question:

1. What does it mean to "know" mathematics for this teacher? (Qs 7,11,17) (Later rated 1-5: 1 = emphasis on procedures; 5 = emphasis on conceptual understanding.)

2. How does this teacher think that children learn mathematics? (Qs 11-13) (Later rated 1-5: 1 = emphasis on knowledge transmission; 5 = emphasis on knowledge construction.)

3. What is the teacher's view of the teacher-student relationship--who are the sources of knowledge, choices, and responsibility for learning in the classroom? (Qs 11-16) (Later rated 1-5: 1 = teacher as the source; 5 = student or students as the sources.)

4. What factors does the teacher see as the source of student ability and diversity in her classroom? (Q 8)

5. What does CGI mean to this teacher? (Q 23)

6. What is this teacher's classroom practice in mathematics? To what extent does the teacher use CGI in her mathematics teaching? (Rated 1-5: 1 = no use; 2 = occasional; 3 = supplemental to other programs; 4 = main basis for mathematics teaching; 5 = total basis.)

7. How does this teacher conceive of subject matter knowledge in mathematics?

A. In her own earlier life? (Qs 3,5)
B. Now, especially in reference to specific research-based information taught in the CGI workshop (i.e., children's strategies for solving addition/subtraction problems and the 11 addition/subtraction word problem types--based on spontaneous mention throughout interview)?
C. What type of knowledge is necessary for teaching first-grade mathematics? (Q 6)

8. & 9. What does the teacher report made it easier (facilitators) or harder (barriers) for her to move to teaching mathematics using CGI? (Qs 18,19,21,22)

10. What results does the teacher report having achieved by using CGI? (Q 17)

11. What is the pattern of change in this teacher's use of CGI? (Qs 9,10)

12. What other mathematics programs does the teacher report using, and how do they relate to CGI? (Qs 11,12)

13. Has the teacher's use of CGI influenced her thinking about and teaching of other subjects? If so, how? (Q 20)

To address these analytic questions, we winnowed through the text of each interview, selecting the sections which seemed to address each analytic question, then aggregating these selections on one card for each question. Using these data, we constructed summaries, codings, and interpretations of each teacher's response to the issues in each question. Thus, the *Hyperqual* program allowed us to create cards, tag text, summarize notes, and develop text files

for each teacher that consisted of a "card" for each analytic question that provided the summary notes and the evidence from the text that addressed that question. In addition, it allowed us to compare this information for each question across all the teachers, which was helpful in visualizing ranges and patterns of responses.

Coding, categorization, and rating of interview data were conducted as an iterative process during the analysis. In other words, we constructed our analytic questions after an initial reading of the interview transcripts and then used each teacher's responses in the interview as evidence for how the teacher responded to each of these questions. As we examined the building pattern of responses, we saw how teachers' responses could be either rated or categorized according to the framing question being considered. For example, teachers' responses to the question, "What does CGI mean to you?" tended to fall into one of three categories--having students solve word problems, using manipulatives to solve problems, or using and building on children's mathematical knowledge in mathematics teaching. Consequently, teachers were placed in one of these three categories. Later, we made an important distinction between the meanings that teachers accorded to CGI, whether they viewed CGI as procedures and techniques to be used or as a group of concepts or a philosophy. Thus, we further divided teachers into those for whom CGI seemed to have a procedural meaning (e.g., using word problems and using manipulatives) and those for whom CGI had a conceptual or philosophical meaning (e.g., that children have mathematical knowledge and teachers should understand and build on that knowledge in their mathematics teaching).

In conducting the *Hyperqual* analysis, we found that we could quantify some of our answers to the 13 guiding questions. For example, teachers' ideas about what it means to know mathematics, drawn from their stated goals for students and their reported evaluation practices, were scored on a conceptual-procedural continuum from 1 to 5, with 1 indicating an emphasis on being able to do procedures and 5 indicating an emphasis on understanding concepts. Teachers' conceptions of how children learn mathematics were also rated on a 1 to 5 continuum, with 1 indicating that the teacher transmits knowledge (i.e., teacher explains, has children practice

skills) and 5 indicating that children construct mathematical knowledge (i.e., children verbalize their mathematical strategies and thinking; children inquire and solve problems on their own).

In response to guiding question three regarding the teacher-student relationship, almost all teachers characterized themselves as "facilitators." However, an important dimension that emerged was the extent to which the students, versus the teacher, were the sources of knowledge, choice, and responsibility for learning in the classroom. Teachers rated higher on the student-teacher relationship dimension (i.e., 5) gave their students choices in the classroom and fostered students' responsibility for their own learning in various ways: such as by encouraging students to write their own mathematics problems, to work as a community, and to develop mathematical strategies and solutions for themselves. These teachers also indicated that the individual student or students as a group served as a primary source or arbiter of mathematical knowledge in their classrooms. Teachers rated lower on this dimension (i.e., 1) gave students little choice of activities, tended to explain or demonstrate a strategy rather than trying to develop or understand students' strategies and solutions, and served as the main source or arbiter of mathematical knowledge in their classroom.

We rated each teacher's use of CGI from 5 to 1, depending on whether the teacher seemed to use it as her only program (5), her main program (4), supplementally (3), occasionally (2), or not at all (1). The evidence that we used to make these ratings came from the detailed description that the teacher gave the interviewer of a typical mathematics lesson on a typical day in her mathematics class, including the kinds of mathematics activities that would occur during mathematics class, the mathematics content and mathematics problems in a typical lesson, the kind of discourse that would typically occur, and the grouping and organization of the students and the classroom.

Finally, we put all the coded responses to these analytic questions on a spreadsheet, as well as all facilitators and barriers mentioned. We did counts and constructed histograms and scatterplots of data to obtain a general idea of relationships. Where appropriate, we computed correlations for quantitative variables and Chi-square tests of association for categorical variables.

Results and Discussion

How Are Teachers Using CGI Knowledge and Ideas?

Three or four years after teachers participated in the month-long CGI workshop, these teachers still think about and use CGI ideas in their classroom practice. From teachers' descriptions of their typical mathematics lessons, we judged that 19 out of the 20 teachers continue to use CGI regularly in their mathematics teaching. But teachers varied widely in their knowledge and use of CGI. We judged that some teachers use CGI ideas only occasionally or supplementally to their regular mathematics program, while others use mainly or only CGI in teaching mathematics. The histogram shown in Figure 1 summarizes the degree of CGI use by these teachers.

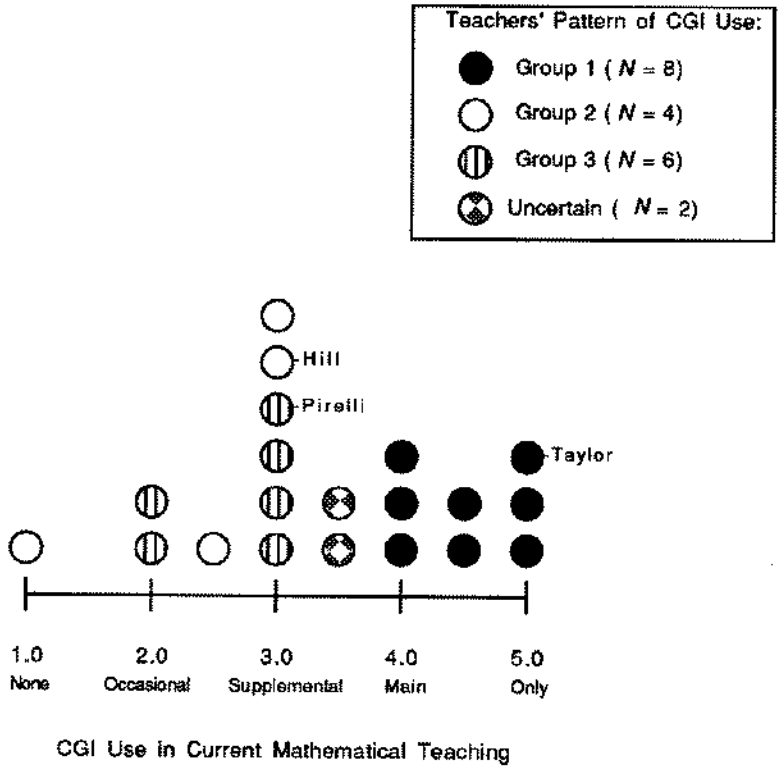

Figure 1. Histogram showing where each teacher was rated on current CGI use and indicating the teacher's reported pattern of use.

These teachers show equally wide ranges on the three dimensions relating to what it means to know mathematics; how children learn mathematics; and who has choice, responsibility, and serves as source and arbiter of knowledge in the teacher-student relationship. Ratings on these dimensions reflect our judgments of teachers' knowledge/beliefs and practices about each of these issues. We do not distinguish between knowledge and beliefs. If ideas about these issues are "taken to be shared beliefs of a significant portion of the research community studying mathematics learning and teaching," then they are considered "knowledge" by that community (Cobb, Yackel, & Wood, 1988). Accordingly, the distinction between knowledge and beliefs becomes blurred. This is a change in stance from that taken by Peterson et al. (1989) who referred to teachers' ideas about these issues as "beliefs."

Teachers' ratings on these three dimensions were highly positively intercorrelated with each other and with ratings of the teacher's degree of CGI use. Table 1 presents these correlations.

Table 1

Correlations Between CGI Use and Teachers' Ratings of Teachers' Knowledge and Reported Practices on Three Dimensions Relating to Mathematics, Learning, and the Teacher-Student Relationship ($N = 20$)

Variable	Mathematics (Procedural-Conceptual)	Learning (Transmission-Construction)	Teacher-Student Relationship and Roles
Learning (Transmission-Construction)	.84		
Teacher-Student Relationship and Roles	.78	.84	
CGI Use	.69	.78	.79

We do not conclude from these correlations that teachers who had this knowledge or these beliefs prior to the CGI workshop were more likely to implement CGI-based instruction. Indeed,

research with these teachers before the CGI workshop and one year later showed that, regardless of teachers' initial knowledge and beliefs prior to the workshop, teachers agreed more with these CGI ideas one year after the workshop (Carpenter et al., 1989). Instead, some teachers' self-descriptions of the changes in their thinking and practice suggested that changes in their thinking about these ideas typically accompanied, rather than preceded, changes in practice. For example, Nancy Gale[3] described her teaching prior to taking the CGI seminar as follows:

> I was real teacher-directed, and I told them exactly what to do, how to solve it. You know, we did the examples on the board and I'd said, "OK, now you do this and now you do that" and followed the textbook and didn't listen to the kids at all . . . I felt like they were learning how to write down answers to numbers, but I didn't know if they really truly understood it.

Now Ms. Gale sees her students as people who can make decisions and construct mathematical knowledge:

> And now . . . I don't tell them how to do anything. I just sort of give the problem and let them come up with their ways, and they do the teaching, and they explain to me how they solved it. And I listen to the kids and I sort of plan my instruction from the kids, rather than from what the textbook says to do next.

Ms. Gale's portrayal to us during the interview of the changes in her thinking and practice corresponds in many ways to the portrait presented by Fennema, Carpenter, and Loef (in press) of the changes in this teacher over the course of four years.

It seemed to us that, for some teachers, the relationship between changes in thought and practice was interactive: As teachers began using some CGI ideas in their teaching, they saw their students solve complex problems and listened to them using sophisticated mathematical thinking. This, in turn, encouraged them to give their students more opportunities to engage in CGI-type contextualized problem solving. Alice Kennet, another teacher who uses CGI as the main basis for her mathematics instruction, described a conversation she had with a second-grade teacher who is at the beginning of this process:

> She came up to me last night, and she said, "Look at these story problems; you have to see these things! Oh my gosh, there are two steps in here and look at the numbers." She was so excited. She said, "I didn't think they could do this. Well, look what they can do!" And I think that's the hook right there--that the kids are going to be a catalyst for change, once we get the teachers to see what

---

[3]All teachers' names in this paper are pseudonyms.

> they're actually doing or thinking about. [CGI gives] kids the opportunity to do that.

What Makes It Easier or More Difficult to Use CGI?

Why did this interactive process of learning from changes in their own students occur with only some of the teachers? Teachers themselves offered many explanations of factors that made it easier or more difficult to change toward CGI-type teaching in mathematics. In response to question 18, 17 of the 20 teachers spontaneously mentioned as an important factor in their own change the availability of time to talk on a regular, extended basis with other teachers who were also using CGI. Such discussions might occur informally within schools or more formally through the CGI discussion groups set up by the sponsors of the program. Not only was time and opportunity to talk with other teachers mentioned as helpful by the greatest number of teachers, but it was also the factor they singled out most frequently as the most important to them. Dorene Ahler, another teacher who bases her mathematics teaching mainly on CGI, gave a typical response:

> I think it's helpful for teachers to talk to one another and to share, and to take some of the mystique out of it. If you're scared to teach something, and you don't have all the answers, to know that that's O.K. . . . I think that made a difference because we compared notes and shared ideas . . . It's always helpful if you can discuss things with other adults, other teachers, rather than being isolated. Sometimes teaching does become so isolated.

Many teachers (15 out of 20) also mentioned the importance of administrative support through money for new materials, permission to deviate from traditional curricula, and recognition of teachers' efforts and expertise, as in the case of Kathy Cole, who said,

> When a local T.V. producer wanted to come out and watch someone doing this new CGI math thing, she [the principal] asked the guy to come into my room to videotape us, which to me was an affirmation from her that she thought what I was doing had value and was worth talking about.

Teachers also identified factors which made it more difficult to move toward CGI-based mathematics teaching. Many teachers felt that time was a significant consideration, with 12 teachers reporting that using CGI takes more class time than traditional text or workbook methods of teaching. An equal number of teachers reported that CGI took more planning time, at least initially. Teachers' expressed need for more time was related to another issue deemed important

by 14 of the teachers interviewed: CGI was not a ready-to-go program with premade sets of problems, activities, and tests, but rather a body of knowledge from research which each teacher had to decide how to use in her particular situation. This characteristic of CGI was a two-edged sword, demanding much of the teachers and making many of them feel very uncertain, especially at first, but at the same time freeing them to adapt CGI ideas to their own teaching style, their students, and their school context. Cecelia Taylor, one of the teachers whose thinking and practice we will describe in more detail later in this paper, expressed this conflict eloquently:

> The first year we all started out, we all kind of felt like we were treading water a lot, . . . you were out in the middle of this lake and you had to kind of swim to the shore that you wanted. It wasn't like you were on a raft, and it was going to hold you up, and you had things right there to draw from. . . . It wasn't as though you were out there with nothing. I mean purely nothing. But again it wasn't a guide like you're going to start your children out at the beginning of the year doing this. That was our choice that we each made. [On the other hand] you are very free to . . . really tailor it to your own specifications, your own teaching style and to explore. It gets you into exploring a lot of other sources that you might not use if you had a teachers' text book and a resource book and that kind of thing, and you just kind of stayed in those little walls. . . . You really are basing it on what the kids do a lot more, because if your text has certain pages to cover on fractions and then you're done, you kind of quit your fractions, [but] maybe your kids wanted to go beyond fourths, and they wanted to go into eighths and sixteenths and whatever. . . . [Many students] are going to go into a lot of areas that they're interested in and into questions related to math concepts that you wouldn't necessarily have touched on otherwise, so it allows for a lot more freedom and exploration and a lot more of being the facilitator and letting the children be the director when you don't have any kind of curriculum that you are sticking to.

While the above teacher-cited factors were mentioned by many of the teachers in the study, they still did not explain why some of these teachers went on to develop CGI as the mainstay of their mathematics teaching while others only used it as a supplement to more traditional curricula.

<u>What Are the Patterns of Change in CGI Use?</u>

In seeking an explanation for these differences, we began to see that we could not categorize teachers simply as high or low users of CGI. Rather, there seemed to be three different groups of teachers, each showing a distinct pattern of change in CGI use:

Group 1 (eight teachers) reported steadily, if often gradually, developing their use of CGI, and now use CGI as the main or only basis for their mathematics teaching.

Group 2 (four teachers) reported having never used CGI more than supplementally or occasionally, and they seemed fairly settled in their current use.

Group 3 (six teachers) reported using CGI more extensively in earlier years but now using it only supplementarily or occasionally; in other words, their CGI use has peaked and diminished.[4]

Grouping teachers in this way enabled us to see some clear differences between teachers in Group 1 and those in Group 2. These can be seen most vividly through case analyses of the ideas and reported practices of two teachers, one from each group. We begin with Cecelia Taylor, a Group 1 teacher, and then we turn to Roberta Hill, a Group 2 teacher.

The Case of a Group 1 Teacher: Cecelia Taylor

Cecelia Taylor has taught first grade in the same Catholic school since she graduated with a degree in elementary education 11 years ago. She says she "did OK" in mathematics classes as a child and took only mathematics for teachers and a mathematics methods course in college. Last year, she taught mathematics to 26 first graders. She is, perhaps, the epitome of those teachers we placed in Group 1. As Ms. Taylor described it, she has gone through a gradual process of change since she took the CGI seminar four years ago:

> In the beginning, I used my textbook a lot; what I did was I just didn't do all the pages and I did more with the pages I did. . . . I would do story problems to warm up and then do some story problem activities. Then maybe I would have them have a sheet of facts in front of them, and I would give a story problem, and I would see if they could find a fact on that page that would represent my story problem. . . . So I used the textbook still, and students still did a considerable amount of computation that first year. Then by the second year, I used probably only half of my textbook at the most and started implementing the story problems a lot more, having students write their own equations a lot more instead of looking for equations that matched and having them invent their own story problems a lot more. . . . By the third year I did not order my textbook anymore and was free to use the money that I would have spent on a textbook on supplies and on manipulative-type things, and the fourth year then the same.

According to Ms. Taylor, a typical day in her mathematics class during the past year went as follows:

> In science during the spring, we went into a lot of space problems, and . . . so we had a whole gimmick of things they [the students] wore and they shrunk down to the size to fit into the space machine, and we took off, and they went out into space. Well, in their exploration we did a set of problems. [For example, one mission] would be to answer questions related to the satellites around the planets in our own solar system, so we would do some sample problems, and we would

---

[4]Two teachers were not included in this part of the analysis because we were unable to judge from the teachers' descriptions of their practice whether they were using it "mainly" or "supplementally" in their mathematics teaching. Because this was a key distinction between Groups 1 and 2, we judge that these two teachers fall somewhere between Groups 1 and 2.

share. I would give the problem orally, and then they would work it out in whatever way they wanted, using a number board or a chalk board or paper or counters or unifix cubes. They were usually told that they could work with a partner or by themselves. Generally the majority of them worked with a partner. . . . So they would all solve the problem. . . . And then I would ask two to four kids to share what their answer was and how they came up with their answer, and then one of them would speak for the partnership. Usually [the class] would come to a consensus, 99% of the time. [If the class didn't agree on an answer] then they had to convince each other. We would go back and forth and say, "Can you come up to the large board in front and show what you did to get your answer?" and . . . we would usually go back to some type of modeling [with manipulatives or drawings]. . . . I wouldn't say, "So, you see that this is the right answer." I would usually say, "So what do you think now?" And they'd say, "Well, that shows us right in front of us, so it has to be that answer. We'll have to see why our answer's different." . . . Sometimes it would take maybe10 minutes for a warm up problem; if it was a little more complicated,10 to 15 minutes, so that was our whole warm up, just one problem. Other times we would do two to three. Then I would give them mission assignments. I would have the same problems on three different pages, and the only difference in the pages was the number size. [Some] who were struggling a little bit with the numbers 50 to 100, I would give numbers under 50 on their paper. And then some that were right around between 40 and 70, and then some that went up to 100 and a little beyond. I wouldn't tell them what they had to take, I would explain to them that Mr. Design had three mission assignments and that he needed all three to be completed . . . "Now you choose which mission assignment you would like to work on." . . . It would happen two ways: Some would come up with their partner, they'd choose their partner first because they knew they would work on the same kind of level, or they were comfortable with that partner. . . . Other kids would come up and say, "I want a C paper," and then they would walk around and they would say, "Who has a C paper and wants to work with me?" . . .

The mission might take two to three days to complete, so each day we would do a little warm up and then they would either start their mission or go back to working on their mission. [For example] there might be 8 to 10 questions on there, and the first question might be, "How many satellites does Jupiter have? How many satellites does Earth have? How many satellites does Saturn have? If all of these satellites were totaled, how many satellites do these planets have?" And then we did it for all the planets.

[Students got their information] from their space book that they had made which had those statistics in. And then they made a graph from that, which they did, I think, mostly in mathematics class; they may have done some of it in science, where they graphed the satellites for each planet. . . .

[When they were done] I would take one group, and it would take us one or two days to go through the 8 to 10 problems. One child would read a problem; then two or three children, like we did in large group, would share their answer and how they got it, and we would come to a consensus by the end of that problem . . . and we would explain and explore different ways of solving each problem. Then the C group would go and do other alternative activities, and then I would take the B group.

Ms. Taylor sees her role as "much more a facilitator rather than an expounder of knowledge." She feels that she allows the students to direct and to share, and she is only there to guide the students, to assist them in choosing who is going to share, and to set the time limits for how long the students will work on something. She believes that her students have had to assume the roles of the teacher and student at the same time. Consequently, they have had to

think a lot more, be willing to share, and be patient listeners. Ms. Taylor thinks she expects a lot of her students, and she feels she has gotten all that she expects and more.

Ms. Taylor's goals for her students in mathematics include:

> Definitely a love for math and not a fear; that's one of the primary things. Then as far as actual content areas, I would like them to have a basic understanding of number, number concepts and the one-to-one correspondence, which most of them come with, but some don't. . . . I want them to have a feeling about, an understanding of addition and subtraction and how they can represent it in symbolism.

To evaluate students' progress toward those goals, she watches how students work on problems in class, the way they use various manipulatives and explain how they get their answers. She listens when they make up word problems for each other because "they would choose their own numbers and story styles, so we learned a lot." She also gives untimed assessments that she makes up herself:

> Generally, I would make the testing situation comfortable enough for them that there were several kids that would ask, "Can we play that game again?" because I called it a game. I talked with them about how important it was for them to do their own work in this situation, and that it was only for the purpose for helping me know where they were going. It was not to compare with each other. It was not for their parents, it was not for anyone else. . . . I would do a variety of things and always some real easy problems so that everybody could do at least some of the problems. But then I would tell them that some of these problems were second- or third-grade problems, and that they weren't expected to know them yet, but I realized some of them were catching on to these things so I wanted to see where they were, so we could [plan for] our next month or two of math class, based on what they were showing me. You know, they felt real comfortable with that and we never discussed results. In fact a lot of times, I didn't even show them the results. I looked at the results, and they didn't.

Actually, she is quite pleased with what is happening with her students. On report cards, she did not give many Ns ("needs improvement") in "effort," "because generally the kids were very interested and very motivated and did not want to stop at the end of math class." Some of her best students were working with percentages and fractions by the end of last year and posing word problems that required parentheses and exponential notation to express them symbolically. On the standardized tests they took in second grade "there was a marked improvement. . . . They definitely did better than they had done in the years I was doing the textbook-type teaching." Some of her first CGI-taught students have reached fourth grade now, and "it was interesting to

see that the fourth-grade scores were also better than they used to be, and that they were quite consistently doing well in the math area." Parents are also pleased: "A lot of parents who struggled with mathematics would say, 'I can't believe Timmy can explain how he got this answer, because I never understood that kind of thing.'"

In response to our question about the meaning of CGI (Q 23), Ms. Taylor said CGI is "building on a child's previously accumulated knowledge. When you are teaching in a CGI mode, you are working with what children know--with their current understanding, and then you are providing experiences in a facilitating way that allows them to explore and build a greater understanding of the concept." She feels CGI has also influenced her teaching in other subjects.

> Now I ask my kids "Why?" questions a lot more often, and "How?" questions. For instance in reading, when someone gets a word that they didn't know, when they're trying to decode it, I not only say, "Great! That's it, that's the word," I will say, "How did you get that word?" and then they listen to each other, and they will use the tactics that someone else did to get a word. [Or] let's say we were going to start this space unit in science, I [used to] say that we were going to talk about space, and then I would share some general facts about space, and it wouldn't be until we'd gotten into it a couple of days before I might ask them what else they know about space; whereas now I will start out in all areas asking, "What do you know about this topic?" So again, it's looking at what they already know.

The Case of a Group 2 Teacher: Roberta Hill

Roberta Hill is a teacher that we placed in Group 2. She says she liked mathematics very much in school "because the answer was always in the book." She took two mathematics theory courses, as well as general mathematics and mathematics methods in college. After Ms. Hill graduated in elementary education, she was a substitute teacher for several years and then taught fourth and third grades for a year each in her present school, a suburban public school near Madison, WI. Then she shifted to first grade, which she has taught for five years now. Last year she had 20 students in her class. She, too, has made some changes in her practice since taking the CGI seminar. The first year, she would begin each class with several word problems then she would have students work independently in their workbooks. The next year, she stopped using her old text because "the materials were just too much rote memorization." Instead, she used a variety of worksheets culled from teacher workshops and other materials. The third year her

school adopted an Addison-Wesley mathematics series, so she was able to base her worksheet packets primarily on that text, while still including activities from other sources, such as *Math Their Way* (1976), that she likes particularly.

Ms. Hill described a typical mathematics class as follows:

> Every day I start out with teacher-directed problem solving . . . . We'll do teacher-directed problems that I do orally, and they use the counters, and they have some *Math Their Way* things worked out so that the counters are varied and interesting to the kids--watermelon counters, little fish counters. . . . I know sometimes some teachers teach everything around a theme. I may relate it once in a while, but not all the time . . . What I'll do is say, "Amanda had seven apples and a friend gave her three more. How many does she have?" Then they'll do the problem solving. When they're done, they'll raise their hand and I'll ask for more than one person to give the answer to keep them on their toes and keep them interested. . . . What I try to do is relate the problems to what we're going to be doing in their independent math workbook. If it's counting, I'll do counting; if it's adding, I'll do adding; if it's trying to teach a new concept like counting up, I'll put the big number up; or if it's missing addends--every time I do it, I try to address what we're going to be doing independently. . . . I actually teach the strategies to help kids figure it out, putting the big number in their head and then counting up or circling the big number on a paper and counting up.
>
> What we'll do, not every time, but maybe three problems out of the day, I'll stop and ask, "Well, how did you solve that problem?" and I'll ask different kids . . . to explain how they came up with their answer. . . . [If a student answers differently than the others,] maybe I'll ask them to explain how they got it, and we can see why it was wrong, or listen to the person who's explaining it so they can see how to get it the correct way.
>
> I would [do] 10 to 12 [word problems], and then I find they get fidgety. They start losing interest, and they're ready to go on. About 10 minutes is about all you can hold their interest. Then I would give them the worksheets. . . . Some will be straight adding and subtracting, and then maybe the back page will have some story problems they have to read and solve. . . . Then they do their independent work and hand it in. . . . Last year I got everything corrected and home on that same day, but this year I do math correcting after school and send it home the next day. . . . [If they finish early] I have learning centers where basically it's their free time to choose an activity that they want to do. There are three different centers back there which I rotate [the students at] each day. One day they may be on the rug, and there they may work with flashcards.
>
> Then I have a listening center. I have tapes made up for story problems, so if they can't read them, they can listen to the tape and write the number sentence that goes with it, and I have counters back there if they need them . . . [and] we have a game center, which includes the computer, [and] counting games, adding games, and a cash box and some *Math Their Way* games, dominos. Some kids don't get to the centers. If they don't finish their independent work by the time lunch comes around, they come back after they finish their lunch and finish it up. [Then] we have a correction time after lunch where I pass out everything that needs to be corrected, and when they're done they get free reading time.

Partway through the year, Ms. Hill divides her students into high and low ability groups, which then work on different topics as the year progresses:

> They start to split more. . . . Last year, I even got to carrying and borrowing with that top group; . . . the majority tend to be better readers, so they can handle their own work. I try to find simpler story problems for the lower group; . . . some were having trouble adding and subtracting. They could master 12, but anything above adding and subtracting to 18, that was difficult for some of the kids.

Ms. Hill sees her role in the classroom as

> teacher directed to begin with. And then it's more, I usually go around the room while they're working independently. If they have questions, I will help them, not give them the answer, but try to figure it out. More of a guidance type thing than giving them the answer or "This is how you do it." You know, "What can you do to figure it out?" And I guess during their free time, it's just keeping the peace.

The students' role is to "have a good understanding of strategies so they know how to deal with a problem . . . and be able to explain how they came up with that answer and why they got it."

At the beginning of the year, Ms. Hill likes "to back up and make sure that everybody's got a good basic foundation and then build on that. Even numbers, counting, one-to-one correspondence, and then start in with harder concepts of adding and subtracting." Her main goals for her students in mathematics are "that they have strategies to help them solve problems. Just to be acquainted with a wide range of different types of problems, not just rote math all the time, although I'm not against that either. They need a little bit of that too." She also wants her students to "feel good about themselves, that they can be successful at math . . . [to] feel like they finished it; they feel successful that way."

In order to evaluate her students' progress in mathematics, Ms. Hill does "more of an informal observation. I stop and ask different kids. I try to get to each child, to see how they're solving it, within maybe a two or three day span. . . . I correct that daily work, too. It's a good way of checking for understanding." She also uses chapter tests from the text and gives timed tests during second semester: "Some are adding, some are adding doubles, some are just straight subtracting." Students do these tests independently, without manipulatives. "I sort of wean them away from the counting chips after a while, and then bring them back out when I do the tens and ones [or] harder ones like that."

Ms. Hill, too, is pleased with what her students are learning. In addition to working on carrying and borrowing, the higher group last year could do "missing addend" problems, like "[Frank] had four marbles, and a friend gave him some, so he had 12. How many more did his friend give him?" Her first group of students had to take the CAT, and "they did better" than previous groups. Like Cecelia Taylor, Roberta Hill reports that parents are often "amazed at what we're doing."

In response to question 23, Ms. Hill defined CGI as "just a way of reaching the child through more of a hands-on approach, using manipulatives, relating abstract concepts to concrete concepts that they make a relationship to, so that they learn strategies to solve word problems better." She "never really thought about" whether CGI has influenced her teaching in other subjects, but thinks "it probably has because you look at it more from the child's point of view, more hands-on type things . . . not so much reading out of the book."

Discussion of the Cases from Groups 1 and 2

Cecelia Taylor and Roberta Hill seem similar in many respects: Neither is a conventional or traditional mathematics teacher; both describe making increased use of story problems and manipulatives in their teaching; and both see themselves as "effective" teachers, in that they report that their students are learning at least the accepted mathematics curriculum for first grade and are scoring well on standardized tests. But in comparing the text of their interviews, we saw some clear differences in their descriptions of their mathematics teaching and in their ideas about mathematics, learning, and students' knowledge and capabilities. These differences characterized those we found between Group 1 and Group 2 teachers on the whole.

Cecelia Taylor's goals for and means of evaluating her students show that she focuses on conceptual understanding in mathematics, while Roberta Hill's goals and means of evaluation are primarily procedural. This reflects a difference on this dimension between Group 1 and Group 2 as shown by the mean scores for what it means to "know mathematics" in Table 2 and by the histogram in Figure 2.

Table 2

Means and Standard Deviations on Three Dimensions for Groups of Teachers

Dimension	Group 1 (*N*=8)		Group 2 (*N*=4)		Group 3 (*N*=6)	
	M	*SD*	*M*	*SD*	*M*	*SD*
Mathematics (Procedural-Conceptual)	4.56	0.73	1.75	0.50	3.25	1.08
Learning (Transmission-Construction)	4.75	0.38	1.88	0.25	3.08	1.50
Teacher-Student Relationship	4.13	0.79	1.50	0.41	2.67	1.21

Ms. Taylor and Ms. Hill seem to have different ideas about how children learn. Ms. Taylor allows students to develop their own strategies for solving problems. Ms. Hill "actually teaches" the strategies, demonstrating a specific procedure such as "circling the big number on a paper and counting up." Again, this difference reflects the differences between the mean scores for Group 1 and Group 2 on teachers' views of mathematics learning in Table 2 and their pattern on this dimension in Figure 2.

These two teachers also have very different views of teacher/student roles and relationships. Ms. Taylor has students work together, so they can teach each other. She encourages them to make up word problems for each other and allows them to choose the level of problem difficulty they want to engage. She rarely verifies or corrects answers, encouraging the class to work out the correct answers by presenting and evaluating mathematical arguments, often supported by manipulatives. Ms. Hill's students, on the other hand, work mostly alone. They do not make up their own problems, nor do they have any choice about which ones they will do. Ms. Hill collects their work, marks it, and returns it to them to correct. Even group discussion of word problems focuses on getting the right answer or watching others to see how they got it. These differences on the teacher-student relationship dimension were typical of the groups as a

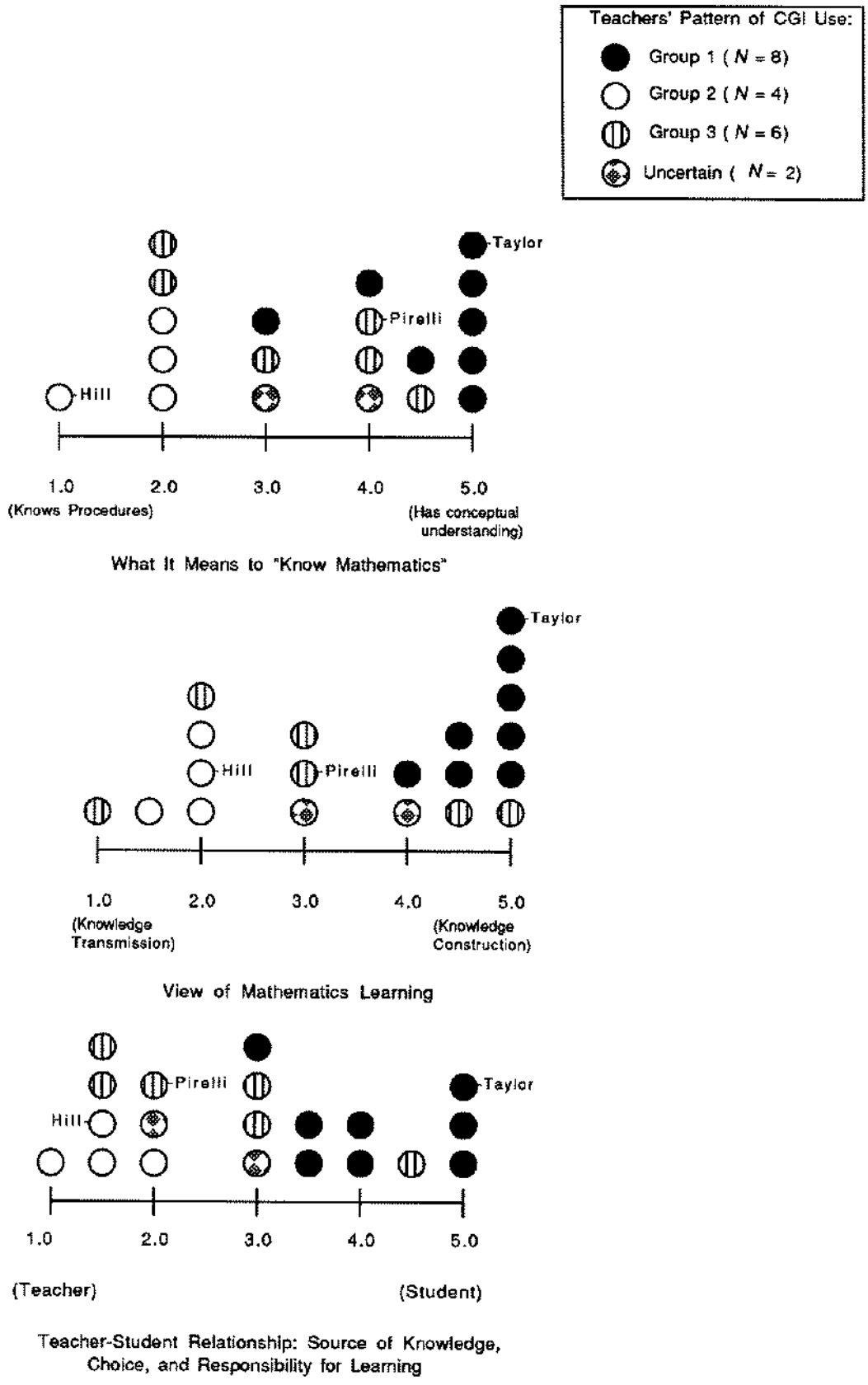

Figure 2. Three histograms showing where each teacher fell on the three key CGI ideas and indicating the teacher's pattern of use.

whole, with Group 1 falling far more toward the student end of the continuum and Group 2 falling far more toward the teacher end. Perhaps most clear cut were the differences in teachers' attitudes toward how knowledge was justified in their classes. Every Group 1 teacher encouraged her students to justify their answers themselves, either through individual sense-making or group discourse, while Group 2 teachers all retained the role of arbiter of correct and incorrect answers.

The teachers see students' reactions to these two types of teaching somewhat differently as well. Ms. Taylor says her students are "very interested and very motivated and [do] not want to stop at the end of math class." They seem unafraid to tackle relatively large numbers and multi-faceted, complex problems. By contrast, Ms. Hill's students seem to be dealing with smaller numbers, and the word problems she cites, although appropriate for first grade, seem relatively straightforward. Ms. Hill sees her students as primarily interested in getting "finished" so they can "feel successful," and she spends a lot of energy trying "to hold their interest." In fact, Group 1 teachers all spontaneously mentioned how much their students like mathematics, while only one of the four teachers in Group 2 said the same.

Like their students, Group 1 teachers also tended to say that they themselves liked mathematics and enjoyed teaching mathematics much more than they had prior to using CGI. More specifically, the three Group 1 teachers who told us they actually disliked and had trouble with mathematics prior to CGI all said that now they enjoy teaching mathematics and have developed a much better understanding of it through using CGI in their classrooms. All three of these Group 1 teachers have gone on to become mentor CGI teachers in the Madison, WI, area. In contrast, three non-Group 1 teachers who reported initial dislike or inability in mathematics reported no such changes in their own attitudes toward and knowledge of mathematics.

Further, Group 1 and Group 2 teachers seemed to have very different ideas about the type of knowledge that elementary teachers need to teach mathematics well to young children. Although we found no significant differences in reported subject matter knowledge or attitudes prior to CGI between teachers in different groups, Group 1 teachers commonly asserted that elementary teachers need substantial subject matter knowledge to teach mathematics well to

young children, while Group 2 teachers consistently asserted that pedagogical knowledge alone was sufficient [$\chi^2(1, N = 12) = 6.00, p < .05$]. Thus, for Group 2 teachers, knowledge of teaching techniques, strategies, and procedures seemed most salient, while the majority of Group 1 teachers felt conceptual understanding of mathematics was also quite important. For example, in response to the question, "What does a teacher need to know to teach math to first graders?" Ms. Taylor replied, "I think first of all she needs to know that children know a lot already." But she also acknowledged that, "they [teachers] should have a definite math strength in their knowledge. . . . A teacher does need to know *why* things work as they do because kids are going to want to know *why* does that work or *how* does that work."

Cecilia Taylor's ideas about the kinds of knowledge teachers need complement the focus she shares with almost all Group 1 teachers on conceptual understanding for her students. Ms. Taylor's ideas about teacher knowledge also seem connected to her ideas of what CGI actually is, and her interpretation of CGI differs substantially from that of Roberta Hill. Ms. Taylor defines CGI in conceptual, almost philosophical, terms as "building on children's already previously accumulated knowledge," a definition similar to those given by all the teachers in Group 1. Ms. Hill, however, sees CGI as a set of procedures, "using manipulatives" and teaching "strategies to solve word problems better," again a definition similar to those of other teachers in her group. This difference between Group 1 and Group 2 teachers on conceptual versus procedural understanding of CGI was statistically significant, [$\chi^2(1, N=12) = 12.00, p < .05$].

Roberta Hill's description of her mathematics teaching practice on a typical day; her proceduralization of CGI; and her ideas about mathematics learning, what it means to know mathematics, and the teacher-student relationship are all strikingly similar to what they were in May 1987 after the end of Ms. Hill's first year using CGI, when Peterson observed her and interviewed her about her practice. Ms. Hill was described as Ms. Hardy in Peterson, Carpenter, and Fennema (1989). Based on students' mathematics achievement scores on the Iowa Test of Basic Skills (ITBS), Peterson et al. found that Ms. Hardy's students had the lowest scores on the ITBS word problem subtest of the 20 CGI teachers' classes, but they performed slightly above the mean on

the ITBS computation subtest. Peterson et al. noted that while Ms. Hardy was the least expert CGI teacher in teaching her students complex problem solving, she was not ineffective in teaching computational skills. Indeed, at that time, she seemed to teach directly for procedural knowledge of addition and subtraction.

It is as though Ms. Hill (Hardy) stopped learning and developing her knowledge and ideas about CGI after the first year of the study, and her knowledge, beliefs, ideas, and practice have remained fairly stable since then. In this way, Roberta Hill seems quite representative of Group 2 teachers, who report having never used CGI more than supplementally or occasionally and who seem fairly settled in their current use. But Roberta Hill seems strikingly different from Cecelia Taylor, whose current thinking and practice seem to have developed considerably from when Peterson observed her at the end of her first year of using CGI. At that time, just as Ms. Taylor portrayed to us, she was still using her textbook and was much more teacher-directed and formalized in her mathematics teaching. Thus, Cecelia Taylor's pattern is likewise representative of her group, Group 1 teachers, who reported steadily, if often gradually, developing their use of CGI, and now use CGI as the main or only basis for their mathematics teaching.

The Case of a Group 3 Teacher: Kathy Pirelli

On the three dimensions in the means table (Table 2) and in the histograms (Figure 2), the six Group 3 teachers appear to fall roughly between the teachers in Groups 1 and 2; however, Group 3's large standard deviation on most measures reveals an important point: Group 3 teachers seem more heterogeneous in their knowledge and reported practices on these dimensions than either Group 1 or Group 2 teachers. Group 3 teachers range from procedural to conceptual (2 to 4.5 on the scale) in their views of mathematics, from transmissive to constructivist (1 to 5) in their ideas about learning, and from quite teacher-directed to highly student-oriented (1.5 to 4.5) in their relationships with students and their views of with whom the responsibility for learning and the authority for knowledge lies. What we found was that while Group 1 and Group 2 teachers' ratings on these dimensions helped us understand the important differences between these groups, Group 3 teachers' ratings on these dimensions did not help clarify our

understanding of this group of teachers. As detailed in the method section, we constructed these scores by rating both the stated knowledge/beliefs and reported practices of the teachers. What we came to realize was that this rating confounded what was the most important characteristic of this group--a seeming incongruity or gap between what Group 3 teachers *say* they think is important in mathematics teaching, and what they report actually *doing* in their classroom practice. We discovered this incongruity by returning to our *Hyperqual* aggregations of the text of the Group 3 teachers' interviews. For some teachers, the contrast between their espoused beliefs and their espoused practice was subtle; for others it was more pronounced, but for all Group 3 teachers an incongruity seemed to exist.

For three of the Group 3 teachers, this gap was apparent to us, but not to them. These teachers saw no incongruity, even as they described their beliefs and then their practices to us in the interview. For example, Judy Simpson, a first-grade teacher in Group 3, said that mathematics is

> more than just adding. I mean, we don't even need to teach them how to add, it seems. You know, there are these little machines that we've got nowadays. . . . I think that you just have to have an understanding of how these numbers work. How is it that we can say two plus two is four, or how is it that we can say two plus three is five, and then three plus two is five?

Yet, several minutes later, when she was asked about her goals for students (Q 7 in the interview protocol), Ms. Simpson's first reply was, "Well, they are going to have to know how to compute." Her subsequent description of her evaluation practices reflected this same computational emphasis; she said she based students' grades "mainly on their performance out of the text, whether it be the chapter tests or just their daily work. I don't evaluate [on] how they handle these real problem situations."

For three of the teachers we studied, though, this incongruity between what they believe about mathematics and mathematics teaching and what they actually do in their classrooms was keenly felt. Kathy Pirelli, who taught 24 first graders in a Madison school last year, was unusually honest in sharing her feelings of conflict with us.

First, Ms. Pirelli described a typical mathematics lesson in her classroom--one which somewhat resembles the lesson described by Ms. Hill of Group 2:

> A typical lesson might be where I would start out and review what we had done the day before, and talk about what we're going to do that day. Then lots of times in our book (Addison-Wesley), they'll have little story problems that you can do, like the problem of the day. I've done those once in a while. I really haven't been real good about doing those. . . . Then I might say to them, "This is what we're going to do for the math period; we're going to learn this today," . . . and they might do some independent work, you know, . . . and then, before the math lesson ends, we would have a story problem time where I would recite some story problems that would relate to whatever concept I was trying to get across. . . . I have them sit; their counters are out, and they're doing it, and I'm walking around and monitoring. [After they discuss the problems] I'll ask, you know, "What was your answer?" and then somebody has a chance to explain how they got the answer, what they did, and then I'll say, "Well, did anybody else do it a different way?" and then somebody else might raise their hand and say, "Well, I did it this way." And during the time that they're saying that, you know, a new strategy might be brought up. Somebody might have done something that I thought was, "Wow! This is a different way to approach it," and I might think, at that time, "Well, this is the right time to bring this up because I think they can handle it," [or] I might not make a big deal of it at all and just say, "Well, that was a different way." I guess it would just depend how ready they were. . . . I've also taken different times where I have actually taught a strategy because nobody brought it up, you know, so I try to cover those. . . . If I see that the majority of students are not getting it, I will certainly go through it. Even a new type of problem where some children get it, and usually, I will do an example with them. I find the kids watching, and then I'll say, "Now it's your turn, on your own."

Yet, Ms. Pirelli is not very satisfied with what she does:

> I always get very excited at the end of the year when I'm going through my files and I think "Why didn't I do that? The year's slipped away."
>
> I know that I don't do [story problems] as much as I did the first years. . . . The kids love it, . . . and [I] usually make a point of doing it, but I don't do it as much as I would like to. I think because, again, it's time. It's knowing what I want to get across to them that day, and also knowing that I want to do some story problems for enrichment, but it's finding the time to fit it all in . . . [Time] really locks you into what you like to do versus what you really can do.
>
> It wasn't until we went to CGI, and our principal said, "Don't worry. You don't have to do the whole book. You can skip pages," that I felt comfortable with that, . . . [but] I do feel that there are certain things I need to get to and that I'm going to have to put [CGI] aside.

Ms. Pirelli sees her students, both their number and their type, as part of the reason she can't teach the way she would like to. "We've always had 24 to 26 students, and I know that in the district where I live, they always have 16 to 18 kids. When you have a smaller group of students, it's easier." She used to have kids make up story problems, and

> they loved to do that; [but] I find my group this year needs more. I have to give an example. Then I find that their stories are so similar to mine that it's not really much of a learning experience as far as them coming up with their own ideas. . . . I guess when we do story problems it takes a lot of time to get the counters out and get them organized, and no matter what you do, three people usually spill, and they have to pick them up. That sounds like a really silly thing, but when you only have ten minutes left, and then they go to gym, and Susie's going to spill her counters like she does every single time. . . . The children we have this year . . . they are sweet little kids, we always have those, but I don't know, they're more immature. I don't know why that is.

Ms. Pirelli wishes she had another adult in the classroom:

> I thought it was so neat in the CGI seminar when we were there, how at the beginning of every year, they would get three or four students at this table to do some story problems to see what level they were at, and I thought to myself, "How can they do that?" You know, you just can't let 20 students have free time play, you know what I mean?

She also feels some pressures from the second-grade teachers in her school, "They were a little disappointed that the students weren't, like I said, really coming back with those facts," and from parents who "have that idea that memorizing those facts is more important than anything in the whole world when it comes to math."

Ms. Pirelli's sense of conflicting priorities in mathematics teaching was also evident in her statements about goals and her means of evaluation.

> I want them to really, really understand the concepts. I don't want them to just walk out and say they know their sums to 10. . . . I want them to have a very good grip on how important adding and subtracting are to everyday life and be able to use those skills. Not just to know them, but to be able to use them, . . . [but] by the end of the year, I think it's important that they have their skills memorized to 10. I guess because by then I feel that if they don't, it's going to slow them down in solving story problems. It will take them a lot longer because they don't know them. Do I feel it's a must? No, I don't; . . . if I had to sit down and say which one is more important, it would be the understanding.
>
> At the end of every math chapter, you know we give them a test. . . . And then I guess, too, you're assessing if you want to look at [the daily workbook exercises]. . . . That is a form of a test to me because . . . I don't tell the students that, but then I can look back on it and think, "This person obviously doesn't understand."
>
> I really like to watch, when I'm saying the story problem, I like to watch what they're doing with their hands. Exactly what they're doing.

She sees her role as teacher and the students' roles in a similarly dualistic fashion:

> I'm going to try to present material to them that is going to probably be brand new and hoping that I can teach that to them and help them to understand that concept. I'm teaching it to them, but I'm also a facilitator because I'm trying to help them learn the concept, although I can't make them learn it. So, I'm someone that's there, and I'm teaching the concept, but they need to be involved; . . . they

> need to be doing. They can't be sitting. For some things, as you're beginning to introduce them, something brand new, there may be a point where they're sitting, but a lot of math at our level, it's got to be doing. . . . So when it's adding, subtracting, doing whatever in math, I'm going to teach it to them, I'm going to present the material very basically, but before I go anywhere with it, they're going to be with me step by step. And that takes time too.

Finally, she explains what CGI means to her:

> A way of teaching math that can be challenging and real exciting. It's requiring students to think through experiences rather than just writing down an answer. They need to use their experiences that they've had and experiences that are becoming new to them and objects around them to be able to figure out what's going on. It's not easy.

Kathy Pirelli describes, almost wistfully, how she would like CGI to affect all her teaching:

> I guess I get into this thing where I'm asking questions and they're simply reciting, and I sometimes stop, and I think, "I know that answer; I know that student knows that answer. I want something more. I want to ask a deeper type of a question." . . . I'm trying to ask more questions that don't have a right answer, "Explain that to me. Why do you feel that way?" or whatever, instead of always just going to the easy one. I think I do that many times during the day, as far as just [going for] the right answer. But somewhere in the back of my head, [I know] that it's so important to ask them, "How did you get that?" I saw those videotapes of students doing things, and it was always "How did you get that?" That relates to everything.

Discussion of Group 3 Teachers

As this last statement shows, Kathy Pirelli seems to have developed her own conceptual understanding of CGI. However, like the other Group 3 teachers, this understanding has somehow remained encysted, somehow separate from the knowledge, beliefs, norms, and values she relies on to resolve the dilemmas she inevitably encounters in her everyday teaching (Ball, 1990b; Lampert, 1985). For all the Group 3 teachers, there seemed to be a striking discontinuity between the goals, beliefs, and ideas they espoused, and the views of mathematics, knowledge, and learning that seemed embedded in the typical mathematics teaching practice they described. Why might such a discontinuity exist for Group 3 teachers?

Kathy Pirelli, for example, feels disempowered in the face of circumstances that she views as preventing her from teaching the way she would like to teach. Although she spontaneously named more different barriers--nine--than any other teacher, the problems that she discussed were shared by more than a few other teachers in this study, and many other teachers were able to cope with them in ways that meshed with their CGI ideas. In particular, the time concerns which

loomed so large for Ms. Pirelli were shared by 14 teachers in the study. However, the Group 1 teachers generally saw their students' knowledge and abilities as a resource for solving this problem. Recall Cecelia Taylor's statement of how her students teach each other. As Cecelia put it, "Now they can [explain] for each other, and it's wonderful because Tommy didn't get this and Billy's explaining it to Tommy. I didn't need to do it; I allow the children to do it for each other." In contrast, Ms. Pirelli sees her students as "sweet" but "immature" and "disorganized"--another barrier that she perceives as making it harder for her to use CGI.

Five of the six Group 3 teachers felt similarly that their students were less suited than others' might be to CGI-type teaching. We questioned whether Group 3's students were actually different from Group 1's students or whether the Group 3 teachers just perceived their students differently. Although we did not collect data on individual teachers' students, we found that the schools in which Group 3 teachers taught served, on average, the same percentage (16% free or reduced lunch) of disadvantaged students as schools in which Group 1 teachers taught and a smaller percentage of disadvantaged students than the schools in which Group 2 teachers taught (33% free or reduced lunch). Nor did Group 3 teachers' classes seem to contain more special needs students than other teachers' classes. Indeed, a Group 1 teacher, Nancy Gale, told us that CGI is

> definitely a lot easier for kids who you might consider really had a hard time in math before. You don't have that many kids who don't understand it anymore. . . . I have quite a few [special needs kids] this year. I have speech and language, and I have an EMR child and an LD child. They were supposed to be sent into the special ed. room for math, but I wanted to keep them in my room and see how it went. They're doing just fine.

Other Group 1 teachers described how CGI is helping ESL, LD, and "slower learning" students in their classes learn mathematics.

In the end, we cannot determine whether the barriers faced by Group 3 teachers are actually greater or whether Group 3 teachers merely perceive them that way; perhaps that is not the important issue. Whether these barriers are "actual" or "perceived," they are real to these teachers and need to be considered by researchers, reformers, and teacher educators if they

hope to help teachers change their practice. Perhaps the most important understanding of Group 3 teachers can be drawn from Kathy Pirelli's words at the end of her interview:

> You need to be there. You need to know what's going on, and you need to feel the pressures that teachers feel to be able to make statements that involve them. I think you need to listen to the people who are trying to do what you're recommending, and that's what you're trying to do here, so I do appreciate it.

Summary and Conclusions

None of the teachers we interviewed seemed to conform to the image of either a "traditional" or "typical" elementary mathematics teacher as described by Romberg and Carpenter (1986) or Peterson, Putnam, Vredevoogd, and Reineke (in press). All the teachers recounted how they use story problems and manipulatives in some way in an attempt to help their students understand mathematics better. From their own descriptions of a typical teaching day, we judged that 19 out of these 20 teachers still use CGI-based practices regularly in teaching mathematics. However, these teachers differed significantly in the extent of their use of CGI. Some teachers used CGI as their main or only program while others used it merely supplementally or occasionally. These wide variations in degree of use appeared among the 10 teachers who participated as the experimental group in the initial large-scale study of CGI four years ago as well as among the10 teachers who participated as the control group of teachers and who then took the same CGI workshop a year later.

When we first sought an explanation for the striking differences in teachers' self-reported degree of CGI use, we found that it was significantly intercorrelated with ratings of teachers' knowledge/beliefs and reported practices related to three important CGI ideas concerning mathematics learning, what it means to know mathematics, and teacher-student relationship/roles. Teachers whom, from their descriptions of classroom practice, we rated as using mainly or only CGI to teach primary mathematics were more likely to display beliefs that (a) children learn mathematics by constructing mathematical knowledge (e.g., problem solutions and strategies) for themselves rather than being taught or given mathematical knowledge by the teacher; (b) knowing mathematics requires the development of conceptual understanding, which these teachers

consider more important than knowledge of computational procedures; and (c) students, rather than the teacher, should have the primary roles and responsibilities in mathematics learning, including choosing the kinds of mathematical problem solving and thinking in which they will engage and serving as sources and arbiters of mathematical knowledge in the classroom. In contrast, teachers whom we rated as using CGI only occasionally or supplementally were more likely to display beliefs that (a) children learn mathematics by the teacher's giving the students mathematical knowledge (e.g., teaching them mathematical strategies and how to find correct answers); (b) knowledge of mathematics involves greater emphasis on knowing computational procedures than on developing conceptual understanding; and (c) the teacher, rather than the students, should have the primary role and responsibilities in mathematics learning, including choosing appropriate problems and mathematical activities for students, demonstrating or teaching mathematical strategies to students, and serving as the primary source and arbiter of mathematical knowledge in the classroom.

For the teachers who use CGI as their main or only program, the relationship between their substantive knowledge and beliefs about these ideas and their mathematics practice seems to be an interactive one. Gradually over three or four years, the teachers' knowledge, beliefs, and mathematics practice have been transformed. These teachers constructed their mathematics practice from the several key CGI ideas--the major one being that children have a great deal of mathematical knowledge and understanding from which the teacher should build and develop mathematics instruction. These teachers' mathematics practices are dynamic, changing, and growing as they learn from their children and as they learn by using CGI.

We came to realize that these teachers--for whom CGI involves substantive ideas and beliefs on which they base their mathematics practice--form one important group of teachers using CGI. These eight teachers (Group 1) show a continuous, developing pattern of use in their mathematics teaching, as well as some use of CGI ideas in their teaching of other subjects (e.g., asking students more "Why?" and "How do you know?" questions, finding out what students know and understand about a subject and then building on that in practice). However, among

those teachers who now use CGI only supplementally or occasionally, we noted two separate patterns of development, so we further differentiated them into Groups 2 and 3.

Group 2 teachers seemed not to have developed their use of CGI over the last three or four years; they reported having never used CGI as a major element in their teaching, and they seem to have settled into their patterns of minimal use or, in one case, of no use. Analysis of the transcripts of these teachers revealed a congruity between these teachers' ideas of what CGI was and their description of how they implemented CGI in their mathematics teaching. In this way, Group 1 and Group 2 teachers were similar. However, while Group 1 teachers' understanding of CGI in thought and practice seemed to be conceptual, Group 2 teachers' understanding of CGI in thought and practice was procedural. Group 2 teachers saw CGI as new and better procedures for teaching primary mathematics--doing more word problems, using manipulatives, teaching children strategies for solving word problems (including teaching students the strategies that other children have been found to use). Group 2 teachers similarly continued to see mathematics primarily as knowing mathematical procedures, to see mathematics learning as the teacher transmitting knowledge of mathematical procedures to students, and to see the teacher, rather than the student, as having the primary role in this process. Because Group 2 teachers thought of CGI as a set of new techniques or procedures, they were able to add these onto their existing mathematics program and current techniques without disturbing or substantively transforming their existing beliefs about mathematics, learning, and the teacher-student relationship. And, indeed, their procedural views on each of these issues were in keeping with the proceduralization of CGI that seemed to come through in these teachers' descriptions of their practices.

It is particularly interesting that Group 2 teachers proceduralized CGI even though the researchers involved (Carpenter, Fennema, and Peterson) did not see CGI as a set of procedures but rather as a set of ideas developed from research about young children's mathematics problem solving. Further, the researchers had made a special attempt to present CGI to teachers *not* as a program or set of procedures but rather as research-based knowledge from which teachers might

draw their own implications for their practice and construct their own use of CGI in their mathematics teaching. Similar findings have been reported by other researchers.[5]

In studies on teaching process approaches to writing to teachers, researchers have observed a tendency for teachers to proceduralize the substantive intervention by adopting and teaching the steps while retaining, essentially unchanged, their conceptual understanding of what it means to be a good writer, how one learns to write, and knowledge and authority relationships between teacher, student, and text (e.g., Florio-Ruane & Lensmire, 1990). As in the present study, Florio-Ruane and Lensmire found that such proceduralization by teachers occurred even though the researchers (teacher educators) themselves had fundamentally different conceptual understandings of these issues on writing, learning, and knowledge-authority relationships, and they expected and intended that the teacher learners would also "understand" and develop these ideas as part of using the procedures. Similarly, although Carpenter, Fennema, and Peterson did suggest to teachers the use of manipulatives, the increased use of word problems, and the use of multiple strategies in problem solving by students, they did not see these as prescriptions or procedures but rather as connected to and deriving from ideas of how students learn and understand problem solving in addition and subtraction, what constitutes mathematical knowledge, and the roles of the teacher and the student in constructing mathematical knowledge.

In contrast to the teachers in Groups 1 and 2, whose espoused ideas about CGI and reported classroom practice seemed congruent, Group 3 teachers expressed conceptually oriented ideas about CGI and about mathematics, learning, and teacher-student roles and relationships that seemed to us incongruent with these teachers' self-described procedurally oriented mathematics practices. In half the cases, the teachers themselves seemed unaware of this discontinuity, but in the remaining cases, the teachers recognized this disparity. In the latter

---

[5]Thank you to Dr. David Cohen for facilitating our thinking during a conversation three years ago in which he suggested to Penelope Peterson the idea of "proceduralizaing a substantive intervention" as a way of describing Ms. Hill's teaching during her first year of using CGI.

cases, the teachers expressed feelings of guilt, conflict, and tension over the recognized disparity between their espoused purposes, knowledge, ideas, and beliefs and their actual practice, but they felt powerless to act to reconcile the difference, due to barriers that they seemed to feel were outside of their control.

The barriers most frequently mentioned by Group 3 teachers included lack of planning time and class time, the type of students they had, the expectations of the next teacher who would have these students, standardized tests that their students would take that assessed computational skills in mathematics, and the fact that there was no "packaged" curriculum for CGI. One Group 3 teacher who was teaching a second/third grade class felt particularly powerless and frustrated because "they" (the researchers) had not made a framework for CGI in second and third grade, and she felt insecure about trying CGI on her own without such a framework. What CGI meant for her was defined rather narrowly in terms of the children's strategies for solving addition/subtraction word problems and the addition/subtraction problem types. She did not seem to have a broader conceptual understanding of CGI as related to ideas of children's construction of mathematical knowledge or the students as sources of mathematical knowledge and as responsible for their own learning. Further, this teacher did not see herself as having the authority, responsibility, or knowledge to develop her own ideas and use of CGI. Indeed, all three Group 3 teachers who expressed a conflict between their espoused beliefs and their described practices seemed to feel that some other individuals, structures, or authorities were somehow either responsible for or kept them from developing and using CGI ideas in their mathematics teaching the way they wanted to.

For these three Group 3 teachers as well as the other three Group 3 teachers, their use of CGI appears to have peaked and diminished. Although earlier in their use of CGI, these teachers seem to have been learning--growing and developing in their knowledge and use of CGI--this may no longer be the case. While the Group 3 teachers still espouse CGI ideas in their views and thinking, their self-described practices seem markedly less CGI-like. Although these teachers reported using CGI extensively in earlier years, they now use it only supplementarily or

occasionally. Given the many barriers they cite and the sense of disempowerment some of them communicate, we are not optimistic about the chances that this downward trajectory will be halted or reversed for many of these teachers.

The Group 3 teachers' story is one of initial enthusiasm for change which did not endure over time or in the face of conflicting concerns. Although this story is not a new one in the field of mathematics education, where reforms have waxed and waned over time and where the innovations of one decade all but disappear from the classrooms of the next, the case of the Group 3 teachers is particularly distressing. The CGI researchers had deliberately attempted to empower teachers by giving them access to research-based knowledge of first-grade children's mathematics learning and encouraging them to use, develop, and transform that knowledge in ways that fit their own beliefs, values, and contexts. Yet, the Group 3 teachers, although initially engaged with the ideas of the reform, apparently were either unable or unwilling to continue to develop and change. These findings led us to wonder what types of support might have enabled these teachers to continue their development of CGI. Particularly, we wondered whether a somewhat greater degree of prescriptiveness or specificity in curriculum would have been helpful to these teachers and, if so, how this greater specificity could be offered without compromising the goal of encouraging teachers to believe in their own knowledge, expertise, and ability to make choices, construct curriculum, and develop their mathematics teaching.

This brings us back to the issue with which we began this paper--whether and how researchers and teacher educators might assist experienced practicing teachers in reforming their classroom practice. Perhaps the present study raises more questions than it answers, for the Group 1 teachers are the success story in the saga of CGI, but the Group 2 and Group 3 teachers are the ones from which researchers and reformers might learn the most. The less-than-success stories are those that leave researchers and reformers with their own persistent dilemmas of practice: How might researchers share their knowledge with teachers in ways that advance the thinking of both and that lead to desired changes in teachers' mathematics teaching and in students' mathematical thinking and problem solving? And how might researchers and reformers

connect with practicing teachers in substantive ways to work on closing the gap between the new visions of mathematics learning and teaching and the current reality?

References

Ball, D. L. (1990a). Reflections and deflections of policy: The case of Carol Turner. *Educational Evaluation and Policy Analysis, 12*, 247-260.

Ball, D. L. (1990b). *With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics* (Craft Paper 90-3). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Baratta-Lorton, M. (1976). *Math Their Way.* Menlo Park, CA: Addison-Wesley.

Bruner, J. (1990). *Acts of meaning.* Cambridge, MA: Harvard University Press.

Carpenter, T. P. (1985). Learning to add and subtract: An exercise in problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 17-40). Hillsdale, NJ: Erlbaum.

Carpenter, T. P., Moser, J., & Romberg, T. A. (Eds.). (1982). *Addition and subtraction: A cognitive perspective.* Hillsdale, NJ: Erlbaum.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499-531.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 255-296). New York: Macmillan.

Cobb, P., Yackel, E., & Wood, T. (1988). Curriculum and teacher development: Psychological and anthropological perspectives. In E. Fennema, T. P. Carpenter, & S. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 92-131). Madison, WI: Wisconsin Center for Education Research.

Cohen, D. (1989). Teaching practice: Plus ça change . . . In P.W. Jackson (Ed.), *Contributing to educational change: Perspectives on research and practice* (pp. 27-84). Berkeley, CA: McCutchan.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*, 311-330.

Confrey, J. (1990). A review of the research on student conceptions in mathematics, science, and programming. In C. B. Cazden (Ed.), *Review of research in education 16* (pp. 3-56). Washington, DC: American Educational Research Association.

Elmore, R., & McLaughlin, M. (1988). *Steady work: Policy, practice, and the reform of American Education.* Santa Monica, CA: Rand Corporation.

Fennema, E., Carpenter, T. P., & Loef, M. (in press). Learning to use children's mathematics thinking: A case study. In C. Maher & R. Davis (Eds.), *Relating schools to reality.* Englewood Cliffs, NJ: Prentice-Hall.

Florio-Ruane, S., & Lensmire, T. J. (1990). Transforming future teachers' ideas about writing instruction. *Journal of Curriculum Studies, 22*, 277-289.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. *Harvard Educational Review, 55*, 178-194.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Mathematical Sciences Education Board and National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

Mathematical Sciences Education Board and National Research Council. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum*. Washington, DC: National Academy Press.

Peterson, P. L. (1990). Doing more in the same amount of time: Cathy Swift. *Educational Evaluation and Policy Analysis, 12*, 261-280.

Peterson, P. L., Carpenter, T., & Fennema, E. (1989). Teachers' knowledge of students' knowledge in mathematics problem solving: Correlational and case analyses. *Journal of Educational Psychology, 81*, 558-569.

Peterson, P. L., Fennema, E., & Carpenter, T. (in press). Using children's mathematical knowledge. In B. Means, C. Chelemer, & M. S. Knapp (Eds.), *Teaching advanced skills to at-risk children*. San Francisco: Jossey-Bass.

Peterson, P. L., Fennema, E., Carpenter, T., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*, 1-40.

Peterson, P. L., Putnam, R. T., & Vredevoogd, J., & Reineke, J. (in press). Profiles of practice: Elementary teachers' views of their mathematics teaching. In W. G. Secada (Ed.), *International Journal of Educational Research* [Special issue on mathematics education reform].

Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives on knowing mathematics in elementary schools. In C. B. Cazden (Ed.), *Review of research in education 16* (pp. 57-150). Washington, DC: American Educational Research Association.

Resnick, L. (1989). Developing mathematical knowledge. *American Psychologist, 44*, 162-169.

Romberg, T. (1990). Evidence which supports NCTM's *Curriculum and evaluation standards for school mathematics. School Science and Mathematics, 90*, 466-479.

Romberg, T. A., & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 850-873). New York: Macmillan.

Romberg, T. A., Harvey, J.G., Moser, J.M., & Montgomery, M.E. (1974). *Developing mathematical processes*. Chicago: Rand McNally & Co.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-22.

Sowder, J. (Ed.). (1989). *Setting a research agenda* (Vol. 5). Reston, VA: Lawrence Erlbaum and the National Council of Teachers of Mathematics.

Steen, L. A. (Ed.). (1990). *On the shoulders of giants: New approaches to numeracy*. Washington, DC: National Academy Press.

Stephens, W. M. (1982). *Mathematical knowledge and school work: A case study of the teaching of developing mathematical processes*. Unpublished doctoral dissertation, University of Wisconsin, Madison.

Wiemers, N. J. (1990). Transformation and accommodation: A case study of Joe Scott. *Educational Evaluation and Policy Analysis, 12*, 281-292.

Wilson, S. M. (1990). A conflict of interests: The case of Mark Black. *Educational Evaluation and Policy Analysis, 12*, 293-310.

Appendix

Interview Protocol

Biographical data

1. Check name and school. What grade are you teaching now? How many children are in your class?

2. I'd like to start out by learning a little about your teaching experience. Can you tell me a little about what else you have taught and where?

3. What is your own educational background?

   P: Highest degree?
   P: What math courses did you take in college?
   P: Do you think they have helped you in teaching math to children?
   P: What other staff development activities have you participated in during the last three years?

4. Do you have a favorite subject to teach, or one that you feel is your strongest? Why?

   P: (If math is not mentioned). How do you feel about teaching math?

5. When you were in school, how did you feel about math? Why?

   P: Describe one teacher in math that you remember well.
   P: Why was he/she so good or bad?

Beliefs

6. What does a teacher need to know to teach math to Xth graders?

   P: Get to subject matter and pedagogical knowledge.
   P: How and where do teachers learn these things?
   P: (Ask only if teacher seems comfortable with this line of questioning). What would you like to know more about in relation to teaching math?

7. What do you think are the most important things for first graders to learn in math?

   P: Try to get to computational, cognitive and affective goals.
   P: Why is each goal important?

8. What do most students know about math when they enter your class?

   P: Where did this knowledge come from?
   P: Is there much diversity in your students' math knowledge when they enter your class? If so, what do you think causes this diversity?

Practice

9. Have you changed your approach to teaching mathematics since you started teaching? Please tell me about that.

   P: Causes for change? Was CGI seminar a factor?

10. (If teacher is using or has used CGI). Has your use of CGI changed since you first started using it after the seminar?

    P: Try to get timeline and causes of development. Look at factors like textbook or workbook use, use of tests, influence of other individuals or programs, etc.

11. On a typical day, describe how you teach math. (Try to find out whether children work alone or in groups, how groups are chosen, what type of discourse occurs, what emphasis is placed on number facts, rote processes, problem investigation, alternative solutions, etc.) GENERAL PROBES: Can you give me an example of that? Why do you do that?

12. What texts and/or other materials do you use in teaching math and why?

13. What proportion of students' time is spent on word problems vs. number facts? Why?

    P: Are word problems harder than other problems for kids?
    P: What type of word problems do you use in teaching? Do you use different types of problems at different times or with different kids? (Does teacher use CGI categories to describe problems? Does teacher have a hierarchical idea of categories? Get examples of problems).

14. How do you see your role as teacher in the math lesson? What should the role of the learners be?

15. How do your students know if they are getting the right answers? (Try to determine discourse and basis for authority in class. Are students encouraged to see if an answer is 'reasonable'?)

16. How is your classroom set up physically? Why?

17. How do you know if your children are learning what you want them to?

    P: How do you know in class?
    P: How do you assess their learning formally?
    P: Why do you use these assessment methods?
    P: Are different assessment methods better for assessing different goals?
    P: Are your students given standardized tests? If so, how have they done on them since you started using CG?

<u>CGI specific questions</u>

18. (If teacher is using CGI). What helped you in implementing CGI in your classroom? What made it more difficult?

    P: Try to get at factors like support from administrators or other teachers, materials, students, schedules, time needed, institutional requirements).

19. (If teacher is using CGI). How have parents reacted to your use of CGI?

20. (If using CGI). Have the ideas of CGI affected your teaching in other subjects? Which subjects and how?

21. Why did you decide to participate in the original CGI study?

22. Have you participated in any study groups or other workshops or follow up activities with CGI?

    P: How have these helped you implement CGI in your teaching?

23. In a few sentences, what does CGI mean to you?