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DATA ANALYSIS STRATEGIES FOR QUASI-EXPERIMENTAL STUDIES WHERE DIFFERENTIAL GROUP AND INDIVIDUAL GROWTH RATES ARE ASSUMED

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Abstract

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: for use in quasi-experimental studies, given that individuals
grow differentially. These strategies are: (1) gains in
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scores, (3) gain scores adjusted for differential growth
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intervention and post-intervention measures was unity,
tionship between the two measures was imperfect.

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Stephen Olejnik[2]

Introduction

Educational researchers are interested in studying individuals who continually change. This interest in naturally changing entities has raised some difficult problems in measurement and analysis. Although considerable research has been devoted to this topic (McNemar, 1958; Lord, 1956, 1958, 1963; Bereiter, 1963; Cronbach & Furby, 1970; Linn & Slinde, 1977), the problems remain unresolved.

Problems related to measuring change exist to varying degrees in all research designs. These issues are less troublesome in experimental studies where the investigator can manipulate the interest variables and observe their effects on other variables. Measuring change is more difficult in quasi-experimental studies because the investigator lacks the freedom to manipulate the variables. This study focuses on issues of change associated with the latter design. Specifically, this study utilizes the non-equivalent control group design (Campbell & Stanley, 1966) where results of one or two pretests are available prior to the investigation.

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[1]This paper is a summary of a 1977 doctoral dissertation of the same title. Complete derivations of the formulas found in this report can be found in that dissertation, Michigan State University.

[2]Stephen Olejnik, former IRT research intern, is a research associate at the University of Pennsylvania.

Campbell and Boruch (1975) have detailed several concerns which may arise in the analysis and interpretaton of quasi-experimental data. Their work focuses on the issue of bias in estimating treatment effects. Although several factors contribute to a biased estimate, the entire problem originates from the fact that without randomization there are likely to be substantial differences between the individuals in their initial status on the outcomes to be assessed. While several strategies have been suggested to consider these differences when estimating a program's effectiveness, Campbell and Boruch argue that these adjustment procedures often cannot eliminate all bias. The magnitude of the bias is related to two issues: (1) specifying appropriate variables on which an adjustment can be made, and (2) selecting an appropriate model in which variables can be used to predict change. The second issue, specifying the appropriate analytic model, was of major concern in this study.

Specification of Analytic Model

Specifying the appropriate analytic model is dependent on how individuals change over time. Several researchers have recently considered the issue of growth models (Campbell, 1971; Kenny, 1975; Bryk & Weisberg, 1977).

The Fan Spread Model

Campbell has been concerned with the relationship between growth rates and estimates of treatment effects. He argues that initial differences on the outcome dimension imply differential growth rates. This selection by maturation is presented in Figure 1; the lines represent group average performance over time. The labels for treatment-control are arbitrary.

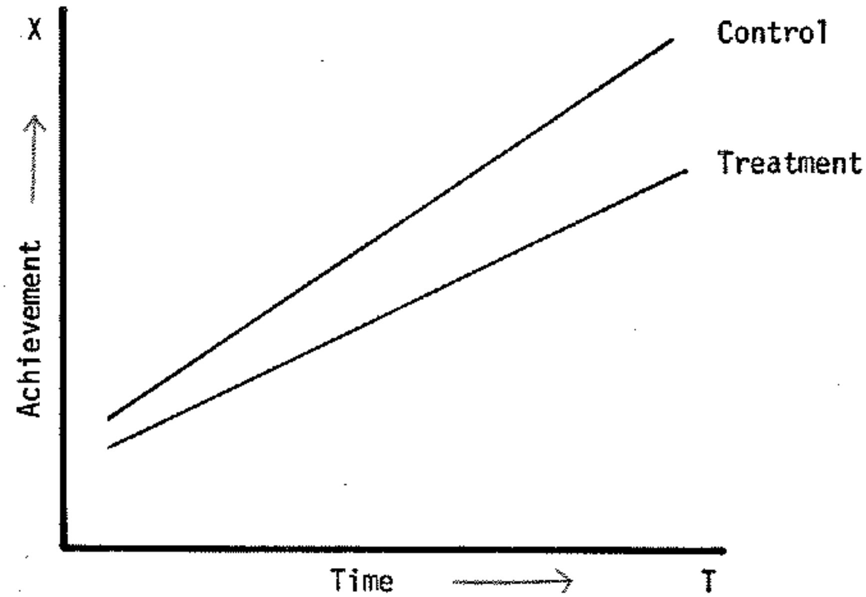

Figure 1. The selection by maturation interaction: increasing mean differences in achievement between comparison groups across time.

Campbell further develops this theory of differential growth rates and labels it the "fan spread hypothesis." The hypothesis states that along with the increasing mean difference between the compared groups, a proportional increase in the variance within the groups occurs. Figure 1 can be modified to reflect the changing variance as shown in Figure 2.

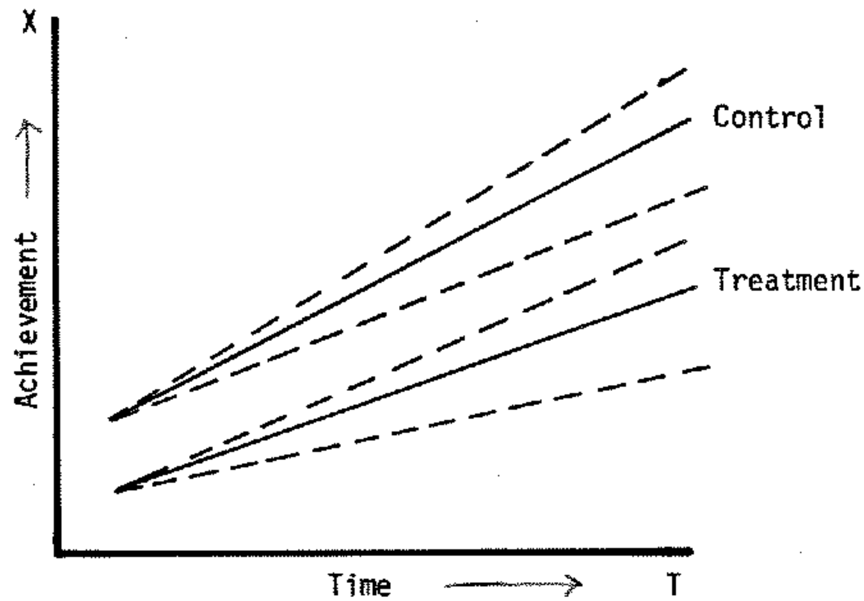

Figure 2. The fan spread hypothesis: increasing mean difference in achievement between comparison groups with a proportional increase in the within-group variability across time.

The broken lines represent the increasing range of achievement scores within the treatment and control groups over time. This relationship between the increasing mean difference and the within-group variance is represented as the formula:

$$\frac{\mu_{x_{p_t}} - \mu_{x_{c_t}}}{\sigma_t} = K$$

where:

$\mu_{x_{p_t}}, \mu_{x_{c_t}}$: are the population means on measure (X) for the program and conrol groups respectively, at time t;

σ_t : is the pooled within-group standard deviation of the outcome measure at time t;

K : is a constant.

Thus the difference between group means relative to the pooled within-group standard deviation remains constant over time. Note that parallel growth patterns between groups may also conform to Campbell's (1971) fan spread model if the within-group variance remains constant across time.

Previous discussions of growth models concentrate on differential growth rates between comparison groups and ignore the issue of differential growth rates within groups. Differences in growth rates within groups can be conceptualized in at least two ways, as presented in Figures 3 and 4.

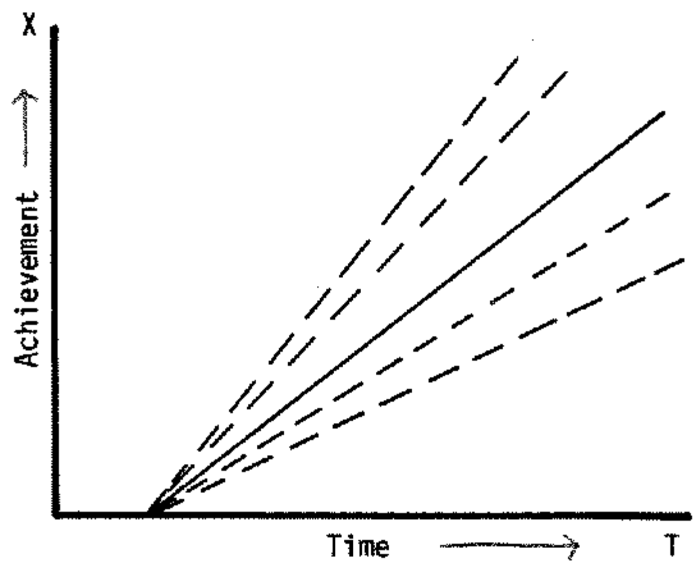

Figure 3. The fan spread hypothesis with the linear model of within-group growth.

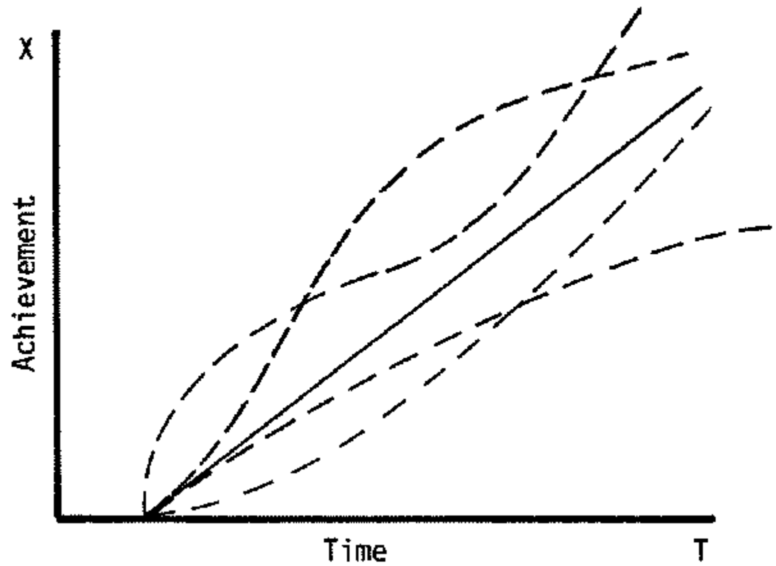

Figure 4. The fan spread hypothesis with a non-linear model of within-group growth.

In each diagram the solid line represents the average growth rate for the group; the broken lines represent individual growth rates. Figure 3 presents within-group growth rates generally associated with the fan spread hypothesis. Within-group growth begins at a common starting point while individual growth rates differ across time. Thus, in any two subsequent points in time, individuals maintain their relative positions within the group.

Figure 4, on the other hand, represents a situation in which the group's mean growth is linear but individual growth is not. Under this model an individual's growth rate may vary over time, i.e., growth may occur in spurts, but group growth may be constant. Both models can result in data conforming to Campbell's fan spread hypothesis, but the implications they have for data analysis and treatment estimation differ substantially.

Given the fan spread model represents a valid conceptualization of how individuals and groups change over time in quasi-experimental studies, Campbell (1971) argues that current analytic strategies are inadequate in adjusting for the differential nature of growth.

<u>The Gains in Standard Scores Strategy</u>

In response to Campbell's argument that current analytic strategies inadequately adjust for the fan spread model, several researchers have proposed new or modified techniques to resolve the differential growth problem. Kenny (1975) argues that given the fan spread model, an appropriate analytic strategy is what he calls standardized gain scores (also referred to as gains in standard scores). The fan spread hypothesis suggests increasing variability within groups across time. Kenny's approach counters this increasing variability by standardizing the pretest and post-test scores using the pooled within-group standard deviation at time 1 and time 2, respectively. The treatment difference can be presented as:

$$\alpha_{GSS}=\mu_{y_p}-\mu_{y_c}-\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})$$

where:

α_{GSS} : is the estimate of the treatment difference estimated by the gains in standard score strategy;

μ_{y_p}, μ_{y_c} : are the population means on the post-treatment measure (Y) for the program and control groups, respectively;

μ_{x_p}, μ_{x_c} : are the population means on the pre-treatment measure (X) for the program and control groups, respectively;

σ_y, σ_x : are the pooled within-group standard deviations of the pre-treatment and post-treatment measures, respectively.

The use of this strategy, like that of raw gain scores, requires that the pre-treatment measure be identical to or a parallel form of the post-treatment measure.

Analysis of Covariance with Estimated True Scores

Another solution to the fan spread model was proposed by Porter & Chibucos (1974). They suggest that the analysis of covariance model is appropriate for the differential growth rate situation if the covariate is perfectly reliable. Given that the covariate is fallible, then analysis of covariance with the estimated true score of the covariate will adequately adjust for the fan spread model. Estimated true score analysis of covariance was originally developed by Porter (1967) as a solution to the single fallible covariate problem.

Using Porter's procedure the program effect can be written as:

$$\alpha_{TS} = \mu_{y_p} - \mu_{y_c} - \frac{\rho_{xy}}{\rho_{xx}} \frac{\sigma_y}{\sigma_x} \left(\mu_{x_p} - \mu_{x_p}\right)$$

where:

α_{TS} : is the estimate of the treatment difference computed by the true score analysis of coveriance strategy;

$\beta_{y.x}$: is the pooled within-group linear regression slope of y on x;

ρ_{xx} : is the reliability coefficient of the covariate and μ_{yp}, μ_{yc}, μ_{xp}, μ_{xc} are as defined previously.

The similarity of this estimate to that of standardized gain scores presented earlier is clearly shown with the following substitution:

$$\beta_{y.x} = \rho_{xy} \frac{\sigma_y}{\sigma_x}$$

The estimate of the treatment difference can now be written as:

$$\alpha_{TS} = \mu_{y_p} - \mu_{y_c} \frac{-\rho_{xy}}{\rho_{xx}} \frac{\sigma_y}{\sigma_x} (\mu_{x_p} - \mu_{x_c})$$

Assuming individuals conform to the fan spread, pre-treatment scores should predict post-treatment scores perfectly, except for measurement errors. Thus the ratio of the correlation between measures and the pre-test reliability is equal to unity, $\frac{\rho_{xy}}{\rho_{xx}} = 1$. The estimate of the program effect provided by true score analysis of covariance and gains in standard scores is the same for fan spread data conforming to the first model of within-group growth. This similarity is only true for the linear growth model for individuals within comparison groups.

When individuals within groups are growing non-linearly the ratio of the correlation between measures and the reliability coefficient of the pretest does not equal unity. The effect estimated by the gains in standard scores and analysis of covariance with estimated true scores is, therefore, different. The two procedures also differ in that the gains in standard scores approach assumes that the correct ratio of the standard deviations is known for the population,while estimated true score analysis of covariance estimates the parameter on the sample data.

The Adjusted Gain Score Approach

Another solution which might be considered to adjust for the fan spread effect is the use of gain scores adjusted for differential group growth. The raw-gain-score strategy assumes that groups change at relatively equal rates, and that the only difference between the groups is the initial status at the point of intervention or observation.

The fan spread model allows that not only do the groups differ in their pre-treatment performance levels, but also that the groups change at different rates. Therefore, simple gain scores could be inappropriate in light of the fan spread model. If the gain scores were adjusted for the differences between the groups' growth rates, an appropriate estimate of the treatment effects might be obtained. Such modification is possible if additional data collected prior to the point of intervention are available.

To facilitate a discussion on development of the modified gain score procedure, Figure 5 details differential achievement growth over time for a hypothetical program and control group without a treatment effect.

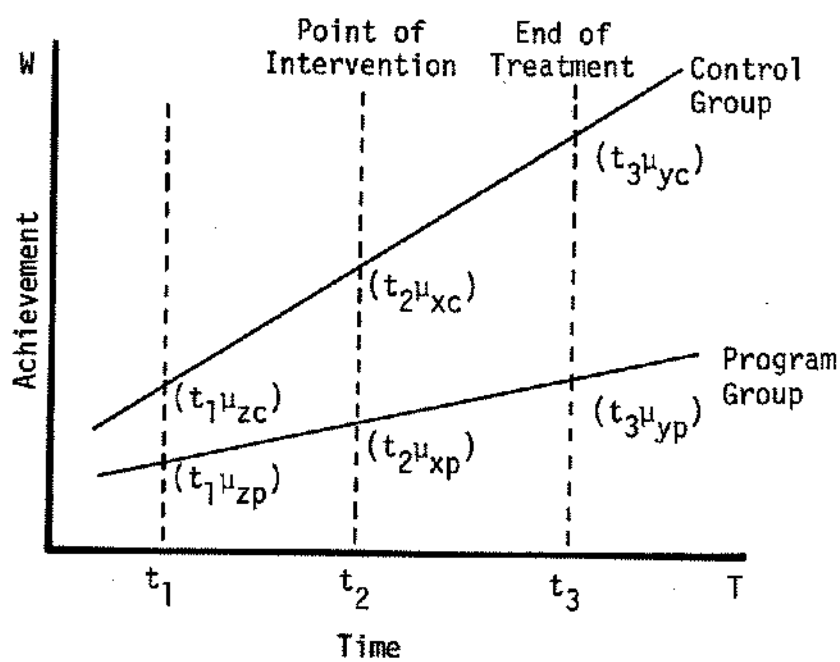

Figure 5. Differential growth rates considered over three points in time.

The horizontal axis T denotes time; the vertical axis W represents achievement. Three points are identified on the time dimension: t_1, t_2, and t_3. The vertical broken line at t_2 indicates the point of intervention, while the dotted lines at t_1 and t_3 represent points in time prior to and at termination of the intervention, respectively. The solid lines represent the linear regression of achievement on time for the program and control group populations. The points at which these regression lines intersect the broken vertical lines represent the average achievement level on the measure administered at time t.

For example, (t_2, μ_{xp}) represents the population mean on measure X for the program group at the time of intervention. These solid lines can be defined in regression equations and used to predict the average group performance at any point in time. If, for example, group performance at t_3 was of interest, the following equations might be used:

$$\mu_{y_p} = a_p + b_p\ (t_3 - t_2)$$

$$\mu_{y_c} = a_c + b_c\ (t_3 - t_2)$$

where:

μ_{y_p}, μ_{y_c} : are the population mean performance on measure (Y) at time t_3 for the treatment and control groups, respectively;

a_p, a_c : are the intercept constants of the regression lines for the treatment and control groups, respectively;

b_p, b_c : are the slopes (rate of growth per unit time) of the regression line predicting achievement from time for the program and control groups, respectively;

$t_3 - t_2$: is the period of intervention.

The difference in average performance of the program and control groups at the termination of the intervention can be determined as:

$$\alpha_{AGS} = \mu_{y_p} - \mu_{y_c} - (a_p - a_c) - \left[(b_p - b_c)\ (t_3 - t_2)\right]. \quad (1)$$

When the intervention has no effect, the equation is:

$$0 = (\mu_{y_p} - \mu_{y_c}) - (a_p - a_c) - \left[(b_p - b_c)\ (t_3 - t_2)\right].$$

Since the intercepts a_p and a_c of the growth curves are the initial achievement levels prior to intervention, then, $a_p - a_c = \mu_{x_p} - \mu_{x_c}$ is the difference in the mean pretest scores of the two groups. With this substitution, the expression (1) becomes:

$$\mu_{y_p} - \mu_{y_c} - (\mu_{x_p} - \mu_{x_c}) - \left[(b_p - b_c)\ (t_3 - t_2)\right] \qquad (2)$$

The first two terms of the equation are identical to raw gain scores that adjust for initial differences in test performance while the second component adjusts for differential growth rates. If the slopes are equal, i.e., the rate of growth is the same for both groups, the second component equals 0 and raw gain scores provide the appropriate adjustment procedure. The fan spread model, however, states that the growth rates are not equivalent and, therefore, an additional adjustment is needed.

The slope of a regression line is defined as the ratio of the change in the vertical axis to the change in the horizontal axis, i.e., $b = \frac{\Delta W}{\Delta T}$. By using the information available before intervention, the growth rate for each group can be estimated. For the program group, the regression slope can be written as: $b_p = \frac{\mu_{x_p} - \mu_{z_p}}{t_2 - t_1}$.

This equation is the ratio of the change in population mean achievement at two points in time prior to intervention with the period of time between testing. Similarly, the regression slope for the control group is:

$$b_p = \frac{\mu_{x_c} - \mu_{z_c}}{t_2 - t_1}$$

With these growth rate estimates, the third term of expression 2 can be written as:

$$\left[(b_p - b_c)\ (t_3 - t_2)\right] = \left[\left(\frac{\mu_{x_p}-\mu_{z_p}}{t_2-t_1} - \frac{\mu_{x_c}-\mu_{z_c}}{t_2-t_1}\right)(t_3 - t_2)\right]$$

$$\left[(b_p - b_c)\ (t_3 - t_2)\right] = \left[\left(\frac{(\mu_{x_p}-\mu_{z_p}) - (\mu_{x_c} - \mu_{z_c})}{t_2 - t_1}\right)(t_3 - t_2)\right] .$$

If the period of time between the first and second testing equals the period of intervention t_2 to t_3, the previous equation can be simplified:

$$\left[(b_p - b_c)\ (t_3 - t_2)\right] = \left[(\mu_{x_p} - \mu_{z_p}) - (\mu_{x_c} - \mu_{z_c})\right] .$$

Thus, the difference in group mean gains prior to intervention can provide an appropriate estimate of the difference in growth rates between program and control groups. The combination of this adjustment for differential growth rates and that for differences in initial performance levels results in the following estimate of treatment effects:

$$\alpha_{AGS} = \mu_{y_p} - \mu_{y_c} - \left[\frac{(\mu_{x_p} - \mu_{z_p}) - (\mu_{x_c} - \mu_{z_c})}{t_2 - t_1}\ (t_3 - t_2)\right]$$

where the terms are as previously defined.

Analysis of Covariance with Multiple Covariates

The strategy presented above requires two assessments prior to intervention. Assuming this pre-treatment information is available, a fourth procedure for data analysis in a quasi-experimental setting conforming to the fan spread hypothesis is analysis of covariance with the two pretests as covariates. Keesling and Wiley (1976) recently suggested a new approach to analysis of covariance using multiple covariates. Their approach, which estimates the treatment effects within groups separately and then compares the magnitude of those effects across groups, may provide a reasonable solution to the question of fallible covariates.

To facilitate comparisons across analytic strategies proposed for solving the fan spread problem, the estimate of a treatment effect can be written as:

$$\alpha_{MAC} = (\mu_{y_p} - \mu_{y_c}) - \frac{\rho_{T_xT_y} - \rho_{T_xT_z}\,\rho_{T_zT_y}}{1 - \rho^2_{Tx\,Tz}}\left(\frac{\sigma_{T_y}}{\sigma_{T_x}}\right)\left(\mu_{x_p} - \mu_{x_c}\right)$$

$$- \frac{\rho_{T_xT_y} - \rho_{T_xT_z}\,\rho_{T_xT_y}}{1 - \rho^2_{Tx\,Tz}}\left(\frac{\sigma_{T_y}}{\sigma_{T_z}}\right)\left(\mu_{z_p} - \mu_{z_c}\right)$$

where:

ρ and σ are the correlation coefficients and standard deviations of the subscripted true variables and;

μ_{yp}, μ_{yc}, μ_{xp}, μ_{xc}, μ_{zp}, μ_{zc} are as defined previously. (The correlation coefficients and standard deviations of the true variables are estimated using replicate measures of the variables involved.)

While this procedure has been demonstrated on an actual data set, there have not been any investigations considering the distribution properties of the test statistic in small samples. Although further study of the Keesling-Wiley procedure is needed before it can be adopted as a competing analytic strategy, it is being considered in this study because the technique appears promising for the future.

Statement of the Problem

Numerous educational research efforts are based on quasi-experimental designs. As a result, researchers in the field often encounter difficult problems in measuring change and estimating treatment effects. Campbell (1971) argues that differential growth rates are not always explicitly recognized in quasi-experimental studies. Furthermore, he argues that traditional analytic strategies fail to consider differential growth patterns when estimating treatment effects. Therefore, estimates of program effectiveness using these strategies will be biased.

Purpose of the Study

The purpose of the study was to compare four procedures in terms of their appropriateness as strategies for data analysis in quasi-experimental studies, given that individuals and groups may grow differentially. The four strategies considered were: (1) gains in standard scores, (2) single covariable analysis of covariance with estimated true scores, (3) gain scores adjusted for differential growth rates, and (4) multiple fallible covariable analysis of covariance. Individual growth was studied both in situations when the correlation between the pre-intervention and post-intervention measures was unity ($\rho = 1$, except for measurement errors), and when the relationship between the two measures was imperfect ($\rho \neq 1$, regardless of measurement errors). This second situation arises when individuals begin to grow at different points in time and grow at different rates, or when individuals grow academically in spurts. (These two situations will be referred to as Condition 1 and Condition 2, respectively.)

The appropriateness of the strategies was based on the effects estimated by each technique and the precision with which each effect was estimated.

Estimating Treatment Effects and Their Standard Errors

The fan spread growth model, as discussed earlier, suggests that concomitant with an increase in mean difference between comparisons groups is a proportional increase in within-group variability. Furthermore, this relationship between the mean differences and pooled standard deviation remains constant across time. Algebraically, this relationship is presented as:

$$\frac{\mu_{x_p} - \mu_{x_c}}{\sigma_x} = \frac{\mu_{y_p} - \mu_{y_c}}{\sigma_y}$$

where the terms are as defined previously.

This representation of the differential growth rate problem indicates that the appropriate adjustment strategy should be:

$$\mu_{y_p} - \mu_{y_c} - \frac{\sigma_y}{\sigma_x} (\mu_{x_p} - \mu_{x_c}).$$

Such an analytic strategy will provide an unbiased estimate of group differences in situations conforming to the fan spread model of growth. Since the definition of the fan spread hypothesis does not include a reference to the nature of the within-group growth pattern, the above approach is appropriate for both Condition 1 ($\rho = 1$) and Condition 2 ($\rho \neq 1$).

Estimation with Gains in Standard Scores

The nature of the hypothesis tested by each analytic strategy is reflected in the respective estimates of group differences. The gains in standard scores approach suggested by Kenny (1975) was shown to estimate group differences as:

$$\alpha_{GSS} = \mu_{y_p} - \mu_{y_c} - \frac{\sigma_y}{\sigma_x}(\mu_{x_p} - \mu_{x_c}).$$

This equation is identical to the adjustment strategy suggested above based on Campbell's (1971) definition of the fan spread model of growth. It is unclear whether the hypothesis was based on manifest or latent variables. If it is defined on the latent true variables, then gains in standard scores uses the ratio of the standard deviations on the observed variables, $\frac{\sigma_y}{\sigma_x}$, when the ratio of the standard deviations of the true variables, $\frac{\sigma_{T_y}}{\sigma_{T_x}}$, is desired. The relationship between the variance of the manifest variables and that of the latent true variables is shown in the following expressions:

$$\sigma^2_x = \sigma^2_{T_x} \rho_{xx}$$

$$\sigma^2_y = \sigma^2_{T_y} \rho_{yy} .$$

The ratio of the standard deviaitons on the manifest variables in terms of the latent true variables can be written as:

$$\frac{\sigma_y}{\sigma_x} = \frac{\sigma_{T_y}\sqrt{\rho_{xx}}}{\sigma_{T_x}\sqrt{\rho_{yy}}} .$$

If the reliability of the pretest equals that of the post-test, $\rho_{xx}=\rho_{yy}$, then the ratio of the observed standard deviation score is appropriate for the latent fan spread model. However, this ratio is an inappropriate adjustment coefficient for the latent fan spread model when the reliability of measures is not equal. Under the manifest fan spread model, the gains in standard scores strategy (as proposed by Kenny) is appropriate whether or not the reliabilities are equal.

While the discussion indicated the appropriate adjustment coefficient for the fan spread model is the ratio of the population standard deviations for the post-test to the pretest, Kenny (1975) uses the sample standard deviations to estimate the ratio. The expected value of the ratio of the sample standard deviations, however, does not equal that of the population standard deviation, $E(S_y/S_x) \neq \sigma_y/\sigma_x$, when the samples are small. The effect estimated using Kenny's technique, therefore, is not the desired one when sample size is small. The gains in standard scores approach is not affected by the relationship among individuals within the comparison groups. For large samples then, the technique can be used to estimate the appropriate effect for both models of within-group growth.

Estimation With True Score Analysis of Covariance

A second solution to the fan spread model proposed earlier is the analysis of covariance model using estimated true scores as the covariate. This approach estimates group differences as:

$$\alpha_{ACTS} = \mu_{y_p} - \mu_{y_c} - \frac{\rho_{xy}}{\rho_{xx}} \frac{\sigma_y}{\sigma_x} (\mu_{x_p} - \mu_{x_c}).$$

This strategy is identical to the adjustment strategy suggested by the fan spread definition, except for the ρ_{xy}/ρ_{xx} ratio, which corrects for measurement errors. If the true relationship between the two measures is perfect as proposed by Condition 1, the ratio of the correlation to the reliability of the covariate will also equal unity. Thus the analysis of covariance model with estimated true scores provides an appropriate adjustment for fan spread in Condition 1.

Previously, a distinction was drawn between the manifest and latent fan spread models. Considering the latent model, the relationship between the adjustment coefficient provided by estimated true score analysis of covariance and the ratio of the latent variable standard deviations can be written as:

$$\frac{\rho_{xy}}{\rho_{xx}} \frac{\sigma_y}{\sigma_x} = \frac{\sigma_{T_y}}{\sigma_{T_x}} .$$

The above expression is true when the reliability of the pretest and post-test is equal for the linear model of within-group growth. If the reliabilities are not equal, then the following relationships show that the appropriate adjustment is still provided by the procedure:

$$\frac{\rho_{xy}}{\rho_{xx}} \frac{\sigma_y}{\sigma_x} = \frac{\rho_{xy}}{\rho_{xx}} \left(\frac{\sigma_{T_x}}{\sqrt{\rho_{xx}}} \right) \Big/ \frac{\sigma_{T_y}}{\rho_{xx}}$$

$$= \frac{\rho_{xy}}{\rho_{xx}} \quad \frac{\sigma_{T_y}}{\sigma_{T_x}} \quad \frac{\sqrt{\rho_{xx}}}{\sqrt{\rho_{yy}}}$$

$$= \frac{\rho_{xy}}{\sqrt{\rho_{xx}\ \rho_{yy}}} \quad \frac{\sigma_{T_y}}{\sigma_{T_x}} .$$

This last expression equals the ratio of the latent standard deviations when the linear model of within-group growth is accurate. If the manifest fan spread model is assumed, then the effect estimated by the true score analysis of covariance strategy is appropriate only when the reliabilities of the pretest and post-test are equal.

Under Condition 2, the true relationship between the pretest and post-test does not equal unity even when measurement errors are corrected. Thus, in this situation the strategy under-adjusts for initial group differences.

Estimation with Adjusted Gain Scores

The third analytic strategy proposed to adjust for differential growth rates in the adjusted gain score strategy. The following illustrates the strategy for estimating group differences:

$$\alpha_{AGS} = (\mu_{p_y}-\mu_{c_y})-(\mu_{p_x}-\mu_{c_x})-\left[(\mu_{p_x}-\mu_{p_z})-(\mu_{c_x}-\mu_{c_z})\right]\frac{t_2-t_1}{t_1-t_0}$$

This expression can be simplified when the period of time between the first and second pretests (t_1-t_0) equals the period of intervention (t_2-t_1):

$$\alpha_{AGS} = (\mu_{y_p}-\mu_{y_c})-2(\mu_{x_p}-\mu_{x_c})+(\mu_{z_p}-\mu_{x_c}).$$

The utility of the effect estimated is demonstrated for the fan spread data by showing that $\alpha_{ASS} = 0$. Assume that the difference between the two group means on the X variable equals some constant (a), $\mu_{x_p}-\mu_{x_c}=a$ and the difference between the group means on the Y measure equals (a + b), where b is any constant, $\mu_{y_p}-\mu_{y_c} = a + b$. For fan spread data and equally distant time points, the difference between the group means on the Z measure would equal a - b; thus, $\mu_{z_p}-\mu_{z_c} = a-b$. The effect estimated using the adjusted gain score strategy can be written as:

$$\begin{aligned}\alpha_{AGS} &= \mu_{y_p}-\mu_{y_c}-2(\mu_{x_p}-\mu_{x_c})+(\mu_{z_p}-\mu_{z_c})\\ &\quad a + b - 2(a) + (a - b)\\ &= 2a - 2a\\ &= 0.\end{aligned}$$

The adjusted gain score procedure does not requre equal time periods between test administration. We can adjust for differences in time periods for the group's using the ratio of time under investigation to the time between the first and second pretests. Since adjusted gain scores are only a function of means and unaffected by measurement errors, the procedure is appropriate for both manifest and latent fan spread models. Finally, the adjusted gain score strategy is not influenced by the model of within-group growth; therefore, it is appropriate for both Condition 1 ($\rho = 1$) and Condition 2 ($\rho \neq 1$) of the fan spread hypothesis.

Estimation with Multiple Fallible Covariates

The final strategy suggested to adjust for differential growth rates between comparison groups is the analysis of covariance model with multiple covariates. The multiple covariates are the double pretest data collected prior to intervention. Following the Keesling-Wiley (1976) procedure an estimate of the group differences is stated as:

$$\alpha_{MAC} = \mu_{y_p} - \mu_{y_c} - \frac{\rho_{T_xT_y} - \rho_{T_xT_z}\rho_{T_zT_y}}{1 - \rho^2_{T_xT_z}} \frac{\sigma_{T_y}}{\sigma_{T_x}} (\mu_{x_p} - \mu_{x_c}) - \frac{\rho_{T_zT_y} - \rho_{T_xT_z}\rho_{T_xT_y}}{1 - \rho^2_{T_xT_z}} \frac{\sigma_{T_y}}{\sigma_{T_z}} (\mu_{z_p} - \mu_{z_c}).$$

On the surface this estimate differs considerably from that suggested by the fan spread model definition. So, the nature of the coefficients must be examined. For Condition 1, the relationship between the test performances was said to be perfect, $\rho = 1$. If we assume this is true, then the following also applies:

$$\rho_{T_xT_y} = \rho_{T_xT_z} = \rho_{T_zT_y} = 1.$$

But the denominator in these adjustment coefficients is $1 - \rho_{T_xT_z}$. If, as Condition 1 suggests, the true relationship between test performances is perfect, then denominators in these coefficients are zero, and the coefficients are undefined. The Keesling-Wiley approach is inappropriate for Condition 1 when the covariates are repeated administrations of the same or parallel forms of the post-test measure.

Condition 2, however, suggests that the relationship between test scores across time is not perfect, $\rho \neq 1$. Under this condition, the adjustment coefficients suggested by the analysis of covariance strategies are defined. It is necessary then to determine whether or not the coefficients provide the appropriate adjustment. To use the Keesling-Wiley procedure under Condition 2 of the fan spread model, the following equality must exist:

$$\frac{\rho_{yx}-\rho_{xz}\rho_{zy}}{1-\rho^2_{xz}}\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})+\frac{\rho_{zy}-\rho_{xz}\rho_{xy}}{1-\rho^2_{xz}}\frac{\sigma_y}{\sigma_z}(\mu_{z_p}-\mu_{z_c})=\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})$$

where:

> correlation coefficients (ρ) and the variances (σ) are expressed in terms of the latent true variables.

The fan spread model defines the difference between the group means on the Z variable as:

$$\mu_{z_p}-\mu_{z_c}=\frac{\sigma_z}{\sigma_x}(\mu_{x_p}-\mu_{x_c}).$$

The Keesling-Wiley coefficient can then be written as:

$$\frac{\rho_{yx}-\rho_{xz}\rho_{zy}}{1-\rho^2_{xz}}\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})+\frac{\rho_{zy}-\rho_{xz}\rho_{xy}}{1-\rho^2_{xz}}\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})$$

$$=\frac{\sigma_y}{\sigma_x}(\mu_{x_p}-\mu_{x_c})\left[\frac{\rho_{yx}-\rho_{xz}\rho_{zy}}{1-\rho^2_{xz}}+\frac{\rho_{zy}-\rho_{xz}\rho_{xy}}{1-\rho^2_{xz}}\right].$$

For the Keesling-Wiley procedure to be appropriate, the third factor in the expression must equal unity:

$$\frac{\rho_{xy}-\rho_{xz}\rho_{zy}}{1-\rho^2_{xz}}+\frac{\rho_{zy}-\rho_{xz}\rho_{xy}}{1-\rho^2_{xz}}=1.$$

This strategy may be rewritten as:

$$\rho_{xy} + \rho_{zy} - \rho_{xz}\ (\rho_{zy} + \rho_{xy}) = 1 - \rho^2_{xz};$$

or

$$(\rho_{xy} + \rho_{zy})\ (1 - \rho_{xz}) = 1 - \rho^2_{xz}\ .$$

Further simplification of the equation is provided by noting that:

$$1 - \rho^2_{xz} = (1 - \rho_{xz})\ (1 + \rho_{xz})\ .$$

Thus, under the second model of within-group growth, the expression $(\rho_{xy} + \rho_{zy}) = (1 + \rho_{xz})$ is essential before the Keesling-Wiley procedure can appropriately adjust for fan spread. This equality only exists when both ρ_{xy} and ρ_{zy} are greater than ρ_{xz}. Since ρ_{zy} involves variables measured at two points farther apart in time than ρ_{xz}, this equality is highly unlikely. Therefore, the Keesling-Wiley procedure is an inappropriate solution to the fan spread model.

Summary

Examination of group difference estimates by the four analytic strategies shows that for Condition 1 ($\rho = 1$), gains in standard scores, analysis of covariances with estimated true scores of the covariate, and adjusted gain scores all estimate the effect of interest. Only the Keesling-Wiley (Note 1) analysis with double pretests as covariates produces the wrong effect estimate. For Condition 2 ($\rho \neq 1$), only the gains in standard scores and adjusted gain scores estimate the desired effect. Thus, researchers can select an analytic strategy in both Conditions 1 and 2 of the fan spread model. Based on these findings, selection of one technique over another might be based on strategy precision.

Precision

The precision of an analytic strategy is defined in terms of the standard error of the contrast, which, in turn, is determined by the fluctuation of the adjusted variable. Therefore, a comparison of standard errors associated with each strategy provides a means of assessing the precision of each technique.

Each analysis strategy considered can be conceptualized in terms of an adjusted dependent measure. When comparing a program group with a control group, the contrast of interest is the difference between the means of the two groups on the adjusted variable. The standard error is, therefore, the square root of the variance of this contrast, $\sqrt{Var(\bar{W}_p - \bar{W}_c)}$. The variance of the contrast is defined as:

$$Var(\bar{W}_p - \bar{W}_c) = Var(\bar{W}_p) + Var(\bar{W}_c) - 2Cov(\bar{W}_p, \bar{W}_c).$$

To determine the standard error of the contrast, both the variance and the covariance of the adjusted means are needed. All analytic strategies considered resemble this form and differ only in the approach used to define the adjusted variable, W. The adjusted variable for both the gain in standard scores and the analysis of covariance model with estimated true scores of the covariate appear as:

$$W = Y - KX$$

where:

X and Y are the pretest and post-test scores, respectively, and K is the adjustment coefficient.

Using the gain in standard scores approch, the adjustment coefficient is the ratio of the standard deviation of post-test to pretest, $K = \frac{s_y}{s_x}$.

The true score analysis of covariance strategy defines the adjustment coefficient as the ratio of the pooled within-group regression slope and the pretest reliability coefficient, $\frac{\beta_{y.x}}{\rho_{xx}}$. The adjusted variable for the adjusted gain score approach is:

$$W = Y - X - \overline{X} + \overline{Z}$$

where:

Z, X, and Y are the first pretest, the second pretest administered just prior to treatment, and the post-test, respectively.

Since the Keesling-Wiley (1976) procedure is inappropriate for the fan spread model, the standard error associated with that technique is not considered.

Table 1 summarizes the standard errors associated with the three competing analytic strategies (see Olejnik, 1977). The formulas presented in Table 1 indicate that the standard errors of the analytic strategies considered are determined by a combination of four components. The first three components involving the variance of the post-test, the variance of the pretest, and the covariance of the pretest and post-test are included in all three standard errors. Standardized gains and analysis of covariance with estimated true scores measure the squared difference between the population means in the fourth component. The fourth component of the adjusted gain score strategy determines the variance of the first pretest and the covariance of the first pretest with the second pretest and the post-test. Since the three standard errors are determined by basically the same components, differences in strategy precision can be explained by differences in the coefficients of the four components.

The first component of each standard error is identical for all three analytic strategies with 1 as the coefficient of the post-test variance

Table 4

The Standard Errors Associated With Gains in Standard Scores, True Score Analysis of Covariance, and Adjusted Gain Scores

Standard error for gains in standard scores:

$$\sqrt{\frac{2}{n}\left[\mathrm{Var}\,(Y)+\left[\left(E\left(\frac{S_y}{S_x}\right)\right)^2+\mathrm{Var}\left(\frac{S_y}{S_x}\right)\right]\mathrm{Var}\,(X)-2E\left(\frac{S_y}{S_x}\right)\mathrm{Cov}\,(X,Y)\right]+(\mu_{xp}-\mu_{xc})^2\ \mathrm{Var}\left(\frac{S_y}{S_x}\right)}$$

Standard error for analysis of covariance for estimated true scores of the covariate:

$$\sqrt{\frac{2}{n}\left[\mathrm{Var}\,(Y)+\left[\frac{\left(E(b_{y\cdot x})\right)^2}{\rho_{xx}^2}+\frac{\mathrm{Var}\,(b_{y\cdot x})}{\rho_{xx}^2}\right]\mathrm{Var}\,(X)-2\,\frac{E(b_{y\cdot x})}{\rho_{xx}}\,\mathrm{Cov}\,(X,Y)\right]+(\mu_{xp}-\mu_{xc})^2\,\frac{\mathrm{Var}\,(b_{y\cdot x})}{\rho_{xx}^2}}$$

Standard error for adjusted gain scores:

$$\sqrt{\frac{2}{n}\left[\mathrm{Var}\,(Y)+\left(1+\frac{3}{n}\right)\mathrm{Var}\,(X)-\left(2+\frac{2}{n}\right)\mathrm{Cov}\,(X,Y)\right]+\frac{2}{n^2}\left[\mathrm{Var}\,(Z)+2\,\mathrm{Cov}\,(Y,Z)-4\,\mathrm{Cov}\,(X,Z)\right]}$$

term. The coefficients of the last three components, however differed considerably. Both gains in standard scores and estimated true score analysis of covariance strategies determine these coefficients using the expected value and variance of their respective adjustment coefficients. But the adjusted gain score approach determines the coefficient of the last three components based solely on sample size.

Tables 2 through 7 were compiled using expected values and variances of adjustment coefficients when the variances of the pretest and post-test are equal. (For complete derivations of these values see Olejnik, 1977.) Expected values and variances were derived for different sample sizes and relationships between the pretest and post-test measures from the theoretical density functions of the adjustment coefficients. The fan spread model assumes increasing variability from pretest to post-test. This assumption has the effect of increasing the expected value and variance of the adjustment coefficients by a factor of the population post-test to pretest variance ratio, $\frac{\sigma_x^2}{\sigma_x^2}$.

Generally, the greater the difference between the two variances, the larger the standard error is for both the gains in standard score approach and the estimated true score analysis of covariance technique. The magnitude of the increase in the standard errors is the same for the two procedures. In comparing the precision associated with these two strategies, the coefficients found in Tables 2 through 7 provide a reasonable basis on which judgments can be made. The adjusted gain score approach is not affected by the fan spread assumption. Therefore, the standard error estimated from the coefficients presented in these tables remains the same regardless of the difference between the variance of the pretest and

Table 2

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .9 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component Cov (X,Y)			Fourth component		
n	Gains in standard scores $\left[E\left(\frac{S_y}{S_x}\right)\right]^2 + \sigma^2_{S_y/S_x}$	Analysis of covariance $\frac{1}{\rho^2_{xx}}\left[E(b_{y\cdot x})^2 + \sigma^2_{b_{y\cdot x}}\right]$	Adjusted gain scores $1 + \frac{3}{n}$	Gains in standard scores $2E\left(\frac{S_y}{S_x}\right)$	Analysis of covariance $2\frac{E(b_{y\cdot x})}{\rho_{xx}}$	Adjusted gain scores $2 + \frac{2}{n}$	Gains in standard scores $\sigma^2_{S_y/S_x}$	Analysis of covariance $\sigma^2_{b_{y\cdot x}}/\rho^2_{xx}$	Adjusted gain scores $\frac{2}{n^2}$
20	1.02228	1.01380	1.15000	2.01100	2.00000	2.10000	.01125	.01380	.00500
40	1.01036	1.00634	1.07500	2.00520	2.00000	2.05000	.00514	.00634	.00125
60	1.00674	1.00411	1.05000	2.00340	2.00000	2.03333	.00334	.00411	.00055
80	1.00486	1.00428	1.03800	2.00240	2.00000	2.02500	.00246	.00428	.00031
100	1.00396	1.00242	1.03000	2.00200	2.00000	2.02000	.00196	.00242	.00020

Table 3

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .8 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component Cov (X,Y)			Fourth component		
n	Gains in standard scores $\left[E\left(\frac{S_y}{S_x}\right)\right]^2 + \sigma^2_{S_y/S_x}$	Analysis of covariance $\frac{1}{\rho^2_{xx}}\left[E(b_{y\cdot x})^2 + \sigma^2_{b_{y\cdot x}}\right]$	Adjusted gain scores $1 + \frac{3}{n}$	Gains in standard scores $2E\left(\frac{S_y}{S_x}\right)$	Analysis of covariance $2\frac{E(b_{y\cdot x})}{\rho_{xx}}$	Adjusted gain scores $2 + \frac{2}{n}$	Gains in standard scores $\sigma^2_{S_y/S_x}$	Analysis of covariance $\sigma^2_{b_{y\cdot x}}/\rho^2_{xx}$	Adjusted gain scores $\frac{2}{n^2}$
20	1.04233	1.03309	1.15000	2.02080	2.00000	2.10000	.02143	.03309	.00500
40	1.01940	1.01520	1.07500	2.00960	2.00000	2.05000	.00978	.01520	.00125
60	1.01255	1.00968	1.05000	2.00620	2.00000	2.03333	.00634	.00968	.00055
80	1.00428	1.00731	1.03800	2.00460	2.00000	2.02500	.00468	.00731	.00031
100	1.00752	1.00580	1.03000	2.00380	2.00000	2.02000	.00372	.00580	.00020

Table 4

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .7 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component (X,Y)			Fourth component		
n	Gains in standard scores $\left[E\left(\frac{S_y}{S_x}\right)\right]^2+\sigma^2_{S_y/S_x}$	Analysis of covariance $\frac{1}{\rho^2_{XX}}\left[E(b_{y\cdot x})^2+\sigma^2_{b_{y\cdot x}}\right]$	Adjusted gain scores $1+\frac{3}{n}$	Gains in standard scores $2E\left(\frac{S_y}{S_x}\right)$	Analysis of covariance $2\,\frac{E\,(b_{y\cdot x})}{\rho_{XX}}$	Adjusted gain scores $2+\frac{2}{n}$	Gains in standard scores $\sigma^2_{S_y/S_x}$	Analysis of covariance $\sigma^2_{b_{y\cdot x}}/\rho^2_{XX}$	Adjusted gain scores $\frac{2}{n^2}$
20	1.05489	1.06122	1.15000	2.02420	2.00000	2.10000	.03049	.06122	.00500
40	1.02753	1.02813	1.07500	2.01360	2.00000	2.05000	.01388	.02813	.00125
60	1.01781	1.01826	1.05000	2.00880	2.00000	2.03333	.00898	.01826	.00056
80	1.01324	1.01352	1.03800	2.00660	2.00000	2.02500	.00663	.01352	.00031
100	1.01048	1.01073	1.03000	2.00520	2.00000	2.02000	.00527	.01073	.00020

Table 5

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .6 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component Cov (X,Y)			Fourth component		
n	Gains in standard scores $\left[E\left(\frac{S_y}{S_x}\right)\right]^2+\sigma^2_{S_y/S_x}$	Analysis of covariance $\frac{1}{\rho^2_{XX}}\left[E(b_{y\cdot x})^2\right]+\sigma^2_{b_{y\cdot x}}$	Adjusted gain scores $1+\frac{3}{n}$	Gains in standard scores $2\,E\left(\frac{S_y}{S_x}\right)$	Analysis of covariance $2\,\frac{E\,(b_{y\cdot x})}{\rho_{XX}}$	Adjusted gain scores $2+\frac{2}{n}$	Gains in standard scores $\sigma^2_{S_y/S_x}$	Analysis of covariance $\sigma^2_{b_{y\cdot x}}/\rho^2_{XX}$	Adjusted gain scores $\frac{2}{n^2}$
20	1.07533	1.10458	1.15000	2.03660	2.00000	2.10000	.03839	.10458	.00500
40	1.03452	1.04805	1.07500	2.01700	2.00000	2.05000	.01745	.04805	.00125
60	1.02252	1.03119	1.05000	2.01120	2.00000	2.03333	.01129	.03119	.00056
80	1.01655	1.02309	1.03800	2.00820	2.00000	2.02500	.00833	.02309	.00031
100	1.01323	1.01833	1.03000	2.00660	2.00000	2.02000	.00662	.01833	.00020

Table 6

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .5 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component Cov (X,Y)			Fourth component		
	Gains in standard scores	Analysis of covariance	Adjusted gain scores	Gains in standard scores	Analysis of covariance	Adjusted gain scores	Gains in standard scores	Analysis of covariance	Adjusted gain scores
n	$\left[E\left(\frac{S_y}{S_x}\right)\right]^2 + \sigma^2_{S_y/S_x}$	$\frac{1}{\rho^2_{XX}}\left[E(b_{y\cdot x})^2 + \sigma^2_{b_{y\cdot x}}\right]$	$1+\frac{3}{n}$	$2E\left(\frac{S_y}{S_x}\right)$	$2\,\frac{E(b_{y\cdot x})}{\rho_{XX}}$	$2+\frac{2}{n}$	$\sigma^2_{S_y/S_x}$	$\sigma^2_{b_{y\cdot x}}/\rho^2_{XX}$	$\frac{2}{n^2}$
20	1.08818	1.17647	1.15000	2.04260	2.00000	2.10000	.04513	.17647	.00500
40	1.04058	1.08108	1.07500	2.02000	2.00000	2.05000	.02048	.08108	.00125
60	1.02628	1.05432	1.05000	2.01300	2.00000	2.03333	.01325	.05432	.00056
80	1.01939	1.03896	1.03800	2.00960	2.00000	2.02500	.00978	.03896	.00031
100	1.01523	1.03093	1.03000	2.00760	2.00000	2.02000	.007613	.03093	.00020

Table 7

Coefficients for the Second, Third, and Fourth Components of the Standard Error Associated With the Three Competing Analytic Strategies When ρ = .4 for the Manifest Variables and the Population Variance of the X and Y Variables Are Equal

	Second component Var (X)			Third component Cov (X,Y)			Fourth component		
	Gains in standard scores	Analysis of covariance	Adjusted gain scores	Gains in standard scores	Analysis of covariance	Adjusted gain scores	Gains in standard scores	Analysis of covariance	Adjusted gain scores
n	$\left[E\left(\frac{S_y}{S_x}\right)\right]^2 + \sigma^2_{S_y/S_x}$	$\frac{1}{\rho^2_{XX}}\left[E(b_{y\cdot x})^2 + \sigma^2_{b_{y\cdot x}}\right]$	$1+\frac{3}{n}$	$2E\left(\frac{S_y}{S_x}\right)$	$2\,\frac{E(b_{y\cdot x})}{\rho_{XX}}$	$2+\frac{2}{n}$	$\sigma^2_{S_y/S_x}$	$\sigma^2_{b_{y\cdot x}}/\rho^2_{XX}$	$\frac{2}{n^2}$
20	1.09883	1.30883	1.15000	2.04760	2.00000	2.10000	.05066	.30883	.00500
40	1.04549	1.14189	1.07500	2.02240	2.00000	2.05000	.02296	.11419	.00125
60	1.02949	1.09211	1.05000	2.01460	2.00000	2.03333	.01485	.09211	.00056
80	1.02179	1.06818	1.03800	2.01080	2.00000	2.02500	.01096	.06818	.00031
100	1.01732	1.05412	1.03000	2.00890	2.00000	2.02000	.00869	.05412	.00020

post-test.

The first three tables (2, 3, and 4) present coefficients for situations when the correlations between measures are high ($\rho \geq .7$). In these situations, only minor differences are apparent between the coefficients defined by the three strategies for the three components. Furthermore, this result is consistent for both small and large samples. Coefficients for the fourth component may demand special attention. When the relationship between the measures is high, the coefficients associated with the fourth component appear very small. For practical purposes these coefficients are essentially zero. Thus, the standard errors for the three strategies under consideration are determined by a combination of the post-test variance, pretest variance, and pretest-post-test covariance components when the two measures are highly related and the pretest and post-test variances are equal.

The second three tables (5, 6, and 7) present coefficients for situations when the correlation between measures is low ($\rho < .7$). The coefficients associated with the second and third components again appear basically the same for each strategy. As sample size increases, the magnitude of the coefficients decreases therefore reducing the standard error and increasing test precision.

The coefficients associated with the fourth component can no longer be judged equal across the three competing strategies. The coefficients for the fourth component of the adjusted gain score strategy are unaffected by the relationship between the measures and thus remain essentially zero. But the coefficients for gains in standard scores and estimated true score analysis of covariance are inversely related to the relationship between the measures. That is, the coefficient increases as the relationship between the measures decreases.

The effect of this relationship is greatest in small samples. As a result of the increase, the fourth component of the standard error formula for these two procedures is no longer zero. Thus, in comparing the precision associated with the three strategies, the adjusted gain score procedure provides the smallest standard error when the relationship between the measures is low.

The coefficients presented in Tables 2 through 7 were determined for situations when the variance of the pretest equalled the variance of the post-test. The fan spread model suggests, however, that variance increases with time. This assumption of the fan spread model does not effect the standard error associated with the adjusted gain score strategy. This technique is influenced only by the size of the sample studied.

The precision associated with the adjusted gain score approach is, therefore, the same as that discussed above. The standard errors of the gains in standard scores and true score analysis of covariance are affected by the fan spread assumption. Further variability predicted by the fan spread model increases both the expected value and the variance of the adjustment coefficient suggested by each procedure. This, in turn, increases the coefficients of the three components discussed previously. Thus the fan spread assumption of increasing variability results in decrease in strategy precision and a larger standard error than when the variability is constant across time.

In comparing the three competing analytic strategies under the fan spread model, then, greater precision is achieved through the adjusted gain score procedure than either gains in standard scores or true score analysis of covariance. Furthermore, the difference in precision increases as the sample size and the relationship between the pretest and post-test measures decreases. Strategy precision will also vary with fan spread, e.g., the greater the fan spread, the greater the difference in precision.

The above discussion comparing the standard errors of the gains in standard scores approach with the estimated true score analysis of covariance approach was based on the false assumption that the standard error associated with the procedure suggested by Kenny (1975) has the form presented in Table 1. In actual practice this is not true.

The computed standard error can be easily derived from a description of how the gains in standard scores were determined. Kenny suggests a two stage process: first determine the pooled standard deviation of the scores at time 1 and time 2; then adjust the respective observed scores with the results from step 1 and take the difference between the two adjusted scores. This difference will be the dependent variable in the analysis of variance model.

It is assumed in this procedure that the adjustment coefficient determined in step 1 is theoretically correct. Assuming the adjustment coefficient computed on the sample data is constant, the same coefficient would be obtained if a second sample were drawn from the population. Based on this assumption the standard error becomes:

$$\sqrt{\frac{2}{n}\ \mathrm{Var}(Y) + \frac{S^2_y}{S^2_x}\ \mathrm{Var}(X) - 2\ \frac{S_y}{S_x}\ \mathrm{Cov}(x,y).}$$

The computed standard error is different from the theoretically correct standard error presented in Table 1 by eliminating all factors involving the variability of the adjustment coefficient. Ignoring the fact that the adjustment coefficient can vary from sample to sample results in an underestimate of the correct standard error. This reduced form of the standard error produces spurious precision and leads to a liberal test of the hypothesis under investigation.

The degree to which Kenny's (1975) procedure is inappropriate depends on the actual variability of the adjustment coefficient. When the relationship between the pretest measure and the post-test measure is high and the sample size is large, the variability of the adjustment coefficient is essentially zero. Under those conditions the procedure suggested by Kenny is likely to be appropriate. In smaller samples, and when the relationship between measures is low, the probability of error associated with Kenny's technique increases.

The second conceptualization of within-group growth proposed for the fan spread model suggested that individuals may not grow linearly. Instead, they may grow at varying rates or in "spurts" across time. As a result, the relationship between the pretest and post-test measures would be less than unity without considering measurement errors. Using this model, two analytic strategies -- gains in standard scores and adjusted gain scores -- achieved the desired estimates. The discussion presented earlier concerning the respective standard errors for these procedures provides the basis on which the selection of one of these approaches can be made.

The coefficients for the adjusted gain score strategy were shown earlier to be influenced only by the sample size under study. Therefore, changing the conceptualization of within-group growth does not affect the direction of the adjusted gain score strategy. The standard error associated with this procedure is the same for the traditional as well as the second conceptualization of the fan spread model. The precision of the strategy using gains in standard scores, however, is influenced by the relationship between the pretest and post-test measures.

The previous discussion concerning this relationship indicated that as the correlation between the measures decreased, the standard error for the gains in standard scores increased. Conceptualizing within-group growth as non-linear reduces the relationship between the pretest and post-test measures beyond that due to errors of measurement. Thus the precision of this procedure is reduced under the second model of the fan spread hypothesis. A comparison of the precision provided by the adjusted gain score approach and the gains in standard scores procedure indicates the former is the more desirable strategy.

In addition, a previous discussion of the computed standard error for the gains in standard scores demonstrated that this technique provided a liberal test of the hypothesis under investigation. The liberalness of this procedure depends on both the sample size and the relationship between the pretest and the post-test measures. Since the second model of the fan spread hypothesis results in a reduced relationship between the measures, the problem of a liberal test of the hypothesis is more acute in this conceptualization than in the traditional approach.

Although both adjusted gain scores and gains in standard scores test the appropriate hypothesis, results indicate that the former procedure provides a more powerful test of the hypothesis and is the more desirable analytic strategy. If data from two pretests were not available, only the gains in standard scores approach tested the appropriate hypothesis. This technique, however, was shown to provide a liberal test of the hypothesis. When this procedure is used, the results of the analysis must be interpreted cautiously. Nevertheless, a correct procedure is available for the standard error developed in this study.

Conclusion

For the fan spread hypothesis under both Condition 1 ($\rho = 1$) and Condition 2 ($\rho \neq 1$), the findings indicated that the most desirable analytic strategy of those considered is adjusted gains. This approach tested the correct hypothesis under both models of the fan spread condition and with greater precision than competing analytic strategies. When only a single pretest performance was available, estimated true score analysis of covariance was shown to be a more desirable strategy than gains in standard scores. This conclusion was limited only to the traditional conceptualization of the fan spread model.

However, when (1) the variance of the measures are equal, (2) the relationship between pretest and post-test is high, and (3) the sample is large, the two procedures estimate the desired effect with equal precision. Finally, when only a single pretest is available and the second model of the fan spread hypothesis is appropriate, only the gains in standard scores procedure estimates the desired effect. Of the four analytic strategies considered in this study, only the multiple covariate analysis of covariance as suggested by Keesling and Wiley (1976) was rejected as an inappropriate technique for any condition of the fan spread hypothesis.

Limitations

The adjusted gain score procedure was presented as an appropriate analytic strategy for situations conforming to the fan spread model. To focus attention on what were considered to be the central points for comparison, some assumptions about the circumstances of application were made. A basic assumption was that it is possible to measure the same individuals repeatedly on the same variable. This assumption may be difficult to meet in a real world setting. In schools, both the administration and teachers require repeated testing to monitor student progress. These tests, however, may not be appropriate for the adjusted gain score approach since they are unlikely to be the same test or a parallel form of the test. With careful, planning, repeated testing of individuals with parallel forms of a test may be possible.

A second assumption was that there are no selection by regression or selection by testing interactions. If these distortions of the group's growth rate affect both groups equally, then an estimate of group differences is not affected. For example, if testing effects equalling (a) units on the X measure exist and this distortion is the same for both groups, then the estimate of the group difference using the adjusted gain score procedure could be written as:

$$\mu_{y_p}-\mu_{y_c}-\left[(\mu_{x_p}+a)-(\mu_{x_c}+a)\right] - \left[(\mu_{x_p}+a-\mu_{z_p})-(\mu_{x_c}+a-\mu_{z_c})\right]$$

$$= \mu_{y_p}-\mu_{y_c}-\left[(\mu_{x_p}-\mu_{x_c}) + (a-a)\right] - \left[(\mu_{x_p}-\mu_{z_p}) - (\mu_{x_c}-\mu_{z_c}) + (a-a).\right]$$

As long as the distortions affect both groups equally, estimates of group differences are still appropriate. The selection by regression or testing interactions refer to the distortions affecting one group to a greater extent than the other group. If this assumption is violated, then the estimated group differences are inappropriate.

Finally, the adjusted gain score strategy was based on the assumption that groups grow in a linear fashion. This assumption is likely to be met in situations involving short time periods. Over extended time periods it seems less likely that a linear model would adequately characterize group changes. The estimated true score analysis of covariance and gains in standard scores strategies also make the same assumption about linear growth. These procedures use data obtained over a shorter period of time than adjusted gain scores and are less likely to violate the assumption. When the assumption is violated, the adjustment provided by the adjusted gain score strategy can be totally inappropriate. Thus, the use of the adjusted gain scores may not be appropriate in situations where the intervention period is extensive.

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